Improving the Noise Immunity of the Modem with the Optimal Finite Signals that do not Cause Intersymbol Interference in a Linear Communication Channel

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Abstract—**A new structure of data transmission system proposed based on the synthesis of digital optimal coherent modem with signals that do not cause intersymbol interference at the output of a band limited communication channel. New analytic expressions for optimal finite signals at the input and output of the low frequency equivalent of the communication channel are presented. The potential noise immunity for the digital optimal coherent modem is calculated. A comparative analysis of noise immunity of the known and new schemes of the optimal coherent modem is carried out. Numerical results and experiment confirm the increased energy efficiency of the proposed modem.**

I. INTRODUCTION

One of the most important tasks for the development of the modern Data Transmission Systems (DTS) is improvement of their noise immunity in the presence of harmful interference and linear distortion in the communication channel (CC) [1-4]. In [5] it is shown, that inter-symbol interference (ISI) plays an increasingly important role in the quality of the signal reception, if data transmission rate is increasing in the linear, band limited CC.

The most known method of dealing with ISI is based on the use of frequency equalizer at the receiver part of DTS [4]. This method is based on the optimization of the reception operator, which provides the maximum likelihood estimation of the signal using the Viterbi algorithm and measured impulse response of the communication channel and a sequence reception as a whole. However, with significant amplitude-frequency fading, such equalization can lead to noise increase at the output of the demodulator and, as a consequence, to a decrease in noise immunity of the DTS. In addition, increasing the number of processed digital messages, the complexity of the implementation of such reception also significantly increases, moreover the noise immunity of DTS in this case can be estimated only approximately.

An alternative approach considered in [8] is associated with the synthesis of such optimal finite signals (OFS), in a form different from the rectangular ones, which do not cause ISI at the output of a band limited CC due to the fact that OFS are designed to match CC impulse response. Thus, a simple optimal OFS symbol-by-symbol reception method is provided based on a matched filter (MF) technology.

This method of OFS transmission and reception allows exact calculation of the noise immunity of the binary transmission systems with multi-position signals, in particular, with quadrature amplitude modulation (QAM).

The International Telecommunication Union (ITU) has developed a number of methods for high-speed data transmission over band limited communication channels, which have found their practical application in modern DTS based on wired technologies and enshrined in the standards: G 992 (ADSL), G 993 (VDSL), etc. [3]. Due to the use of a number of narrow-band filters in the transmission and a set of filters matched with them at the reception, this technology has a number of advantages over other technologies.

Thus, the use of narrow-band shaping filters allows to synthesize optimal finite signals (OFS) for them, which do not cause ISI at their outputs, which does not require the introduction of guard periods, and, consequently, leads to higher efficiency of DTS. The advantages of DTS with narrowband filters and OFS matched with it, has caused an increased interest of various researchers. Therefore, the problem of developing new modems with narrow-band signals and increased spectral efficiency, considered in this paper, is relevant.

The purpose of the present study is to accurately quantify the noise immunity of DTS with OFS that do not cause ISI, using two options for the implementation of the low-frequency equivalent of a coherent modem containing channel filters with non-multiple and multiple poles. The second option is considered for the first time.

II. DATA TRANSMISSION SYSTEMS

The report considers the DTS, the which block-diagram is shown in Fig. 1, where the following notation is used:

- PSTF – pulse-shaping transmission filter;
- BPF – bandpass filters of the modem, which define model of the bandlimited CC;
- CRC – carrier recovery circuit;

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- CFRC – clock frequency recovery circuit; A/D – threshold comparator (decision device). The principle of operation of this DTS is obvious and does not require a detailed description.

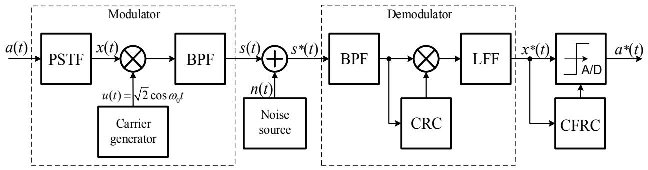

Fig. 1. Model of data transmission system with coherent modem

Analysis and implementation of DTS with coherent digital modems is often carried out on a simpler equivalent model in the baseband (low-frequency equivalent) [8].

The block diagram of the coherent modem's low-frequency equivalent (LFE), on the basis of which the characteristics of DTS are studied, is shown in Fig. 2.

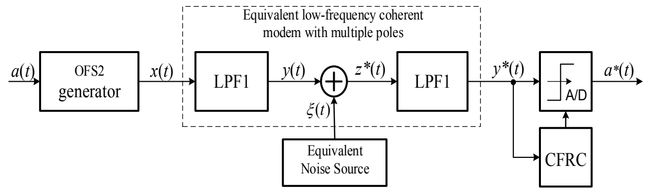

Fig.2. Coherent modem's low-frequency equivalent model

The characteristics of low-pass filters (LPF) of linear LFE CC can be described either by impulse response or by a complex transmission coefficient (transfer function).

In binary transmission systems, the term symbol is equivalent to a bit - a binary signal of rectangular form with duration T, values $a(t) = \pm 1,\ 0 \le t \le T$, the symbol rate is equivalent to the bit rate $V = 1/T$ (bit/s).

III. OFS SYNTHESIS PROBLEM DEFINITION

Suppose that in the model of Fig. 2 the transfer function $k_n(j\omega)$ of the order n CC LFE, including the transmission and reception LPFs, is known.

Then, in accordance with the work [8], the synthesis of OFS $x(t)$ with a spectrum $S_x(j\omega)$ that does not cause ISI at the output of this CC is based on the maximization (for all $S_x(j\omega)$) of the following functional

$$J = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(j\omega) k_n(j\omega) \Phi_n(-j\omega, T) d\omega = y(T/2) \quad (1)$$

under the following restrictions:

1) the values of the signal $y(t)$ at the output of the CC and its derivatives up to the order n are equal to zero at the ends of the symbol interval T;

2) the signal $x(t)$ energy E_x is fixed.

In (1) $y(T/2)$ - is the response of the CC in the center of the symbol interval, $\Phi_n(j\omega, T)$ - signal function

$$\Phi_n(j\omega, T) = e^{-j\omega T/2}\left\{1 - 2\sum_{k=0}^{n-1}\mu_k \omega^k \cos(\omega T/2 - k\pi/2)\right\}, \quad (2)$$

where μ_k, $k = \overline{0, n-1}$, are the Lagrange multipliers of the solution of the extreme problem (1).

Synthesis of CC OFS at the input $x(t)$ and output $y(t)$ performed on the basis of the variational calculus methods in accordance to criterion (1). As a result, the desired signals are defined as follows [8]

$$\underset{-\infty \le t \le T}{x(t)} = \varepsilon\left\{q\left(\frac{T}{2} - t\right) - \sum_{k=0}^{n-1}\mu_k\left[\frac{d^k q(T-t)}{dt^k} + (-1)^k \frac{d^k q(-t)}{dt^k}\right]\right\},$$
$$\underset{-\infty \le t \le T}{y(t)} = \varepsilon\left\{G\left(\frac{T}{2} - t\right) - \sum_{k=0}^{n-1}\mu_k\left[\frac{d^k G(T-t)}{dt^k} + (-1)^k \frac{d^k G(-t)}{dt^k}\right]\right\}, \quad (3)$$

where $q(t)$ - the impulse reaction of the CC, $G(t)$ - channel function equal to the correlation function of the impulse response $q(t)$, ε^2 - constant, determined from the ratio

$$\varepsilon^2 = \frac{2\pi E_x}{\int_{-\infty}^{\infty} k_n^2(\omega)\Phi_n^2(\omega, T) d\omega}. \quad (4)$$

Signals in (3) correspond to the spectra of the OFS at the input and output of CC

$$S_x(j\omega) = \varepsilon k_n(-j\omega)\Phi_n(j\omega, T),$$
$$S_y(j\omega) = \varepsilon k_n^2(\omega)\Phi_n(j\omega, T). \quad (5)$$

IV. PECULIARITIES OF THE LAGRANGE MULTIPLIERS CALCULATION FOR DIFFERENT MODELS OF COMMUNICATION CHANNEL

The signals in (3) will be finite on the symbol interval T only when the Lagrange multipliers in (2) and (3) satisfy the given n constraints imposed in (1) and are determined by the specific type of impulse reaction of the CC. Thus, for a linear CC whose characteristic equation has n different real roots (poles) α_k, $k = \overline{1, n}$, the impulse response is [6]

$$q(t) = \sum_{k=1}^{n} c_k e^{-\alpha_k t},\ t \ge 0, \quad (6)$$

where c_k, $k = \overline{1, n}$ - constants determined from the n initial conditions.

In this case, as shown in [11], Lagrange multipliers are found from the solution of the following system of linear algebraic equations

$$1-\sum_{i=0}^{n-1}\mu_i\alpha_k^i\left(\mathrm{e}^{-\alpha_k T/2}+(-1)^i\,\mathrm{e}^{\alpha_k T/2}\right)=0,\quad k=\overline{1,n} \qquad (7)$$

In another case, when the characteristic equation CC has real roots α_k and root number k has multiplicity m_k (total number of roots taking into account their multiplicity is equal to n), the impulse reaction of such CC is represented as [6]

$$q(t)=\sum_{r=0}^{m_k-1}\sum_k c_{kr}t^r e^{-\alpha_k t},\, t\geq 0 \qquad (8)$$

Then, after a series of calculations, the following system of linear algebraic equations is obtained for Lagrange multipliers [11]

$$\frac{d^r}{d\alpha_k^r}\left\{1-\sum_{i=0}^{n-1}\mu_i\alpha_k^i\left(\mathrm{e}^{-\alpha_k T/2}+(-1)^i\,\mathrm{e}^{\alpha_k T/2}\right)\right\}=0,$$
$$k=1,2,...;\ r=\overline{0,m_k-1}. \qquad (9)$$

V. Characteristics of Optimum Finite Signals Reception Noise Immunity

Let us proceed to the analysis of noise immunity of DTS with coherent modem for different models of CC LFE and different methods of the OFS optimal reception. Let's define that the equivalent source of interference in the model in Fig. 2 produces additive white Gaussian noise (AWGN) $\xi(t)$ with single-sided power spectral density (PSD) G_0.

In this case, taking into account the fact that the optimal signals $y(t)$ are finite at intervals of multiples T and do not cause ISI, their optimal reception in the presence of AWGN is possible on the basis of matched filtering (MF) with impulse response [10]: $q_{SF}(t)=y(T-t), 0\leq t\leq T$, and signal/noise ratio (SNR): $h_{SF}^2=2E_y/G_0$.

Given power transmission ratio (PTR) $k_E(V)=E_y/E_x$ SNR is to mind [9]

$$h_{SF}^2=\frac{2E_x}{G_0}k_E(V). \qquad (10)$$

With equal probability messages and AWGN, the average error probability is [9]

$$p_e=0.5erfc\left(\frac{h_{SF}}{\sqrt{2}}\right)=0.5erfc\left(\sqrt{\frac{E_x}{G_0}\cdot k_E(V)}\right), \qquad (11)$$

where

$$erfc(\rho)=\frac{2}{\sqrt{\pi}}\int_\rho^\infty \mathrm{e}^{-\lambda^2}\,d\lambda.$$

To obtain numerical results, we take as a CC LFE model - first order channel (CC1) with a complex transmission function

$$k_1(j\omega)=\frac{1}{(1+j\omega/\alpha_1)}, \qquad (12)$$

where $\alpha_1=\Delta\omega_\gamma=2\pi F_\gamma$ is the bandwidth of CC1at the 3 dB attenuation level.

A. OFS1 Transmission Over CC1 Noise Immunity

Let's consider the LFE of the optimal coherent modem shown in Fig. 2, in which CC1 there is a transmission LPF with a transfer function $k_1(j\omega)$ from (12), and reception LPF (MF1) is the filter matched with OFS1 $y_1(t)$ defined by (3) subject to (12).

Table I shows the main characteristics of DTS with CC1, where PTR is the power transmission ratio.

TABLE I. MAIN CHARACTERISTICS OF OPTIMAL DTS WITH CC1

Impulse response	$q_1(t)=\alpha_1 e^{-\alpha_1 t},\, t\geq 0$
Channel function	$G_1(t)=0.5\alpha_1 e^{-\alpha_1\lvert t\rvert}$
Lagrange coefficient	$\mu_0=1/2\,\mathrm{ch}(b_1)$, $b_1=\alpha_1/2V$
Energy-transfer coefficient	$k_{1,E}=0.5\left(1-\frac{2b_1}{\mathrm{sh}(2b_1)}\right)$

Fig. 3 shows shapes and spectra of optimum finite signals of the first type (OFS1) at the input of a coherent filter (MF1).

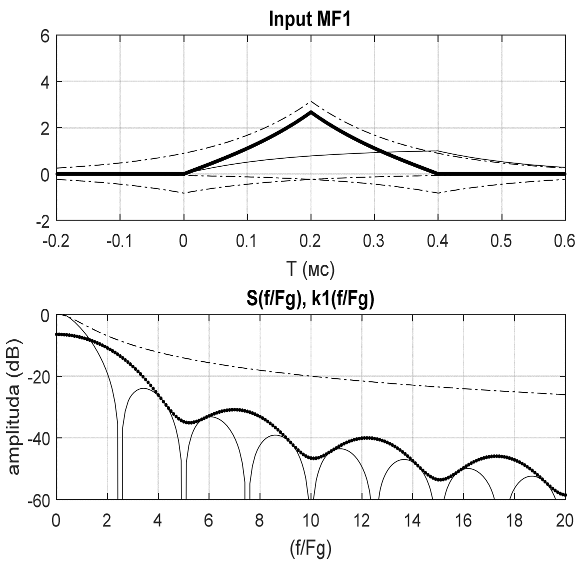

Fig. 3. Shape and spectra of OFS1 at the MF1 input

Fig. 6 shows on the left (lower curve) the dependence of the TPR $k_{1,E}$ on the bit rate V at the boundary frequency LPF1, equal to $F_\gamma=1$ kHz; right graph (upper curve) shows the dependence of the average error probability from V, obtained on the basis of (11) taking into account $k_{1,E}$.

B. OFS2 Transmission Over CC2 Noise Immunity

Analyzing the DTS model in Fig. 1, we may conclude that in the real case, the modulator and demodulator have identical bandpass filters (BPF), which in Fig. 2 correspond to identical LFE1 transmission and reception with transfer functions of (12). But in this case, CC LFE end-to-end transfer function is determined by the following product of transfer functions

$$k_2(j\omega) = k_1(j\omega)\cdot k_1(j\omega) = \frac{1}{\left[(1+j\omega/\alpha_2)\right]^2}. \qquad (13)$$

which is characterized by a double pole, and the value α_2 is determined so as to provide a weakening of the module (13) at the level of 3 dB. Hence, we find: $\alpha_2 = \alpha_1/\sqrt{2^{1/2}-1}$.

A new implementation of the LPE optimal coherent modem is shown in Fig. 4.

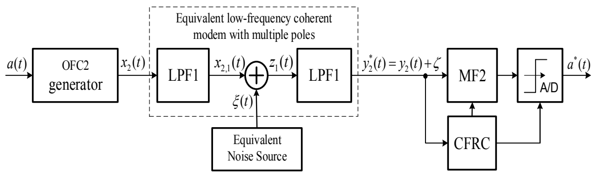

Fig. 4. Low-frequency equivalent of optimal modem with multiple poles

Now the problem of synthesis of OFS2 should be solved on the basis of relations in (3), with a new impulse response $q_2(t)$, as the Fourier transform of (13), and a new channel function $G_2(t)$, as a correlation function of $q_2(t)$, and Lagrange multipliers are sought from the ratio (9).

A specific of the model in Fig. 4, in contrast to the model in Fig. 2, is that here at the input of a coherent filter MF2(CФ2) there is a mixture of OFS2 $y_2(t)$ and colored noise $\zeta(t)$ with following PSD

$$G_\zeta(\omega) = 0.5\mathrm{G}_0\, k_1^2(\omega,\alpha_2) = 0.5\mathrm{G}_0\, k_2(\omega) = \frac{0.5\mathrm{G}_0}{\left[1+(\omega/\alpha_2)^2\right]}. \qquad (14)$$

Table II shows characteristics for CC2 with multiple poles.

TABLE II. MAIN CHARACTERISTICS OF DTS WITH CC2

Impulse response	$q_2(t) = \alpha_2^2 t\, \mathrm{e}^{-\alpha_2 t},\ t \ge 0$
Channel function	$G_2(t) = \left(\left\lvert\frac{t}{\alpha_2}\right\rvert + 1\right)\frac{\alpha_2\,\mathrm{e}^{-\alpha_2\lvert t\rvert}}{4}$
Lagrange coefficient	$\mu_0 = \frac{\mathrm{sh}(b_2)+(b_2)\mathrm{ch}(b_2)}{2b_2+\mathrm{sh}(2b_2)}$ $\mu_1 = \frac{1}{2V}\cdot\frac{\mathrm{sh}(b_2)}{2b_2+\mathrm{sh}(2b_2)},\ b_2 = \alpha_2/2V$

According to [10] SNR at the MF2 output is defined as:

$$h_{SF}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{S_{y_2}^2(\omega)d\omega}{G_\zeta(\omega)}.$$

Given (5) and (14), the ratio for SNR is represented as

$$h_{SF}^2 = \frac{2\varepsilon^2}{G_0}\frac{1}{2\pi}\int_{-\infty}^{\infty}k_2^3(\omega,\alpha_2)\Phi_2^2(\omega,T)d\omega.$$

Considering now equation (4), SNR at the MF2 output is represented as

$$h_{2,SF}^2 = \frac{2E_x}{G_0}\frac{\int_{-\infty}^{\infty}k_2^3(\omega)\Phi_2^2(\omega,T)d\omega}{\int_{-\infty}^{\infty}k_2^2(\omega)\Phi_2^2(\omega,T)d\omega} = \frac{2E_x}{G_0}k_{2,E}(V), \qquad (15)$$

where

$$\Phi_2^2(\omega,T) = \left[1-2\left(\mu_0\cos(\omega T/2)+\mu_1\omega\sin(\omega T/2)\right)\right]^2 \qquad (16)$$

The shapes and spectra of the optimal finite signals of the second type (OFS2) at the input of the matched filter (MF2) are shown in Fig. 5.

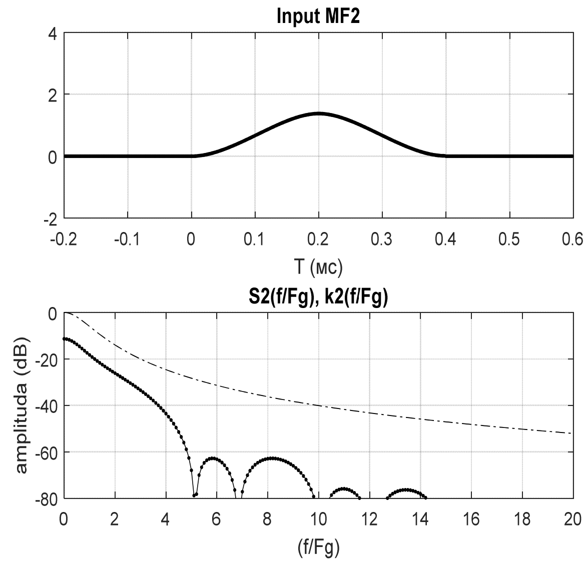

Fig. 5. Shape and spectra OFS2 at the entrance MF2

Fig. 6 shows on the left (upper curve) the dependence of the PTR $k_{2,E}$ on the bit rate V at the LPF2 boundary frequency, equal to $F_\gamma = 1$ kHz; on the right graph of this figure (lower curve) shows the dependence of the average error probability from V, obtained on the basis of (11) taking into account $k_{2,E}$.

The obtained dependences characterize the potential noise immunity of the DTS with a coherent modem, built on the basis of optimal finite signals that do not cause intersymbol interference at the input of the DTS demodulator decision device.

Moreover, as it is shown at Fig. 6, a new structure of optimal LFE modem with multiple poles (Fig. 4) has better energy characteristics comparing to the known one.

This is also confirmed by the dependence of the potential noise immunity for the compared models, shown in Fig. 7.

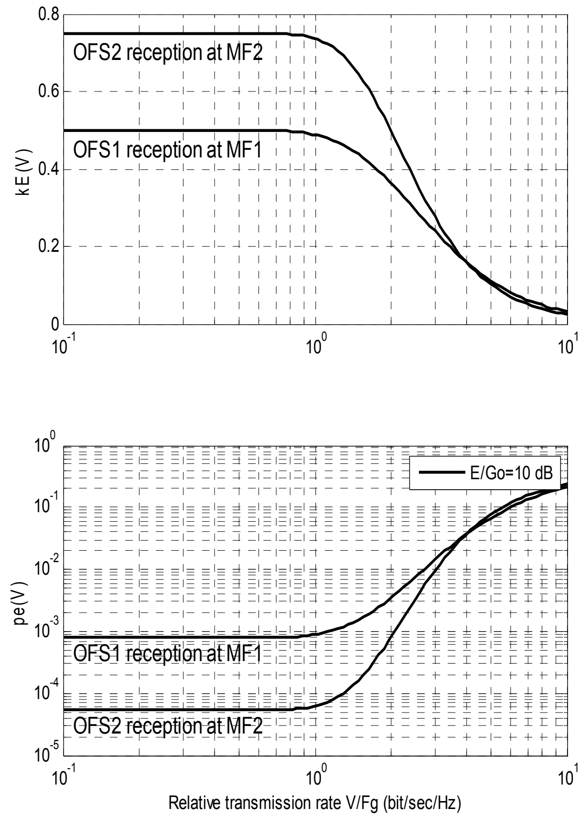

Fig. 6. PTR and average error probability dependences on bit rate

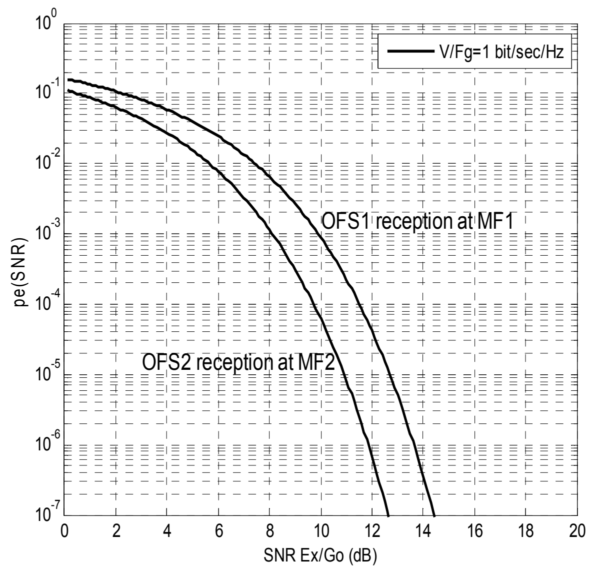

Fig. 7. Average error probability dependencies on the signal-to-noise ratio

VI. EXPERIMENT

Fig. 8 shows experimental results of the study (in MATLAB) of the data transmission system with optimal finite signals (OFS2) that do not cause intersymbol interference at the output of a bandlimited communication channel with the transfer function (13).

Signals in different sections of the DTS are shown (top to bottom):

1) the original binary message at the input of the PSTF (Fig. 1);
2) the optimal finite signal at the output of the PSTF (for comparison, a rectangular signal is also shown);
3) the optimal modulated signal at the output of the modulator;
4) optimal modulated signal at the output of the bandlimited communication channel;
5) the signals at the output of the demodulator (blue graph – signal envelope at the input of the demodulator, dash-dotted - the response of the demodulator to a signal of a rectangular shape, a continuous chart - OFS);
6) the signals at the output of the coherent filter of the demodulator.

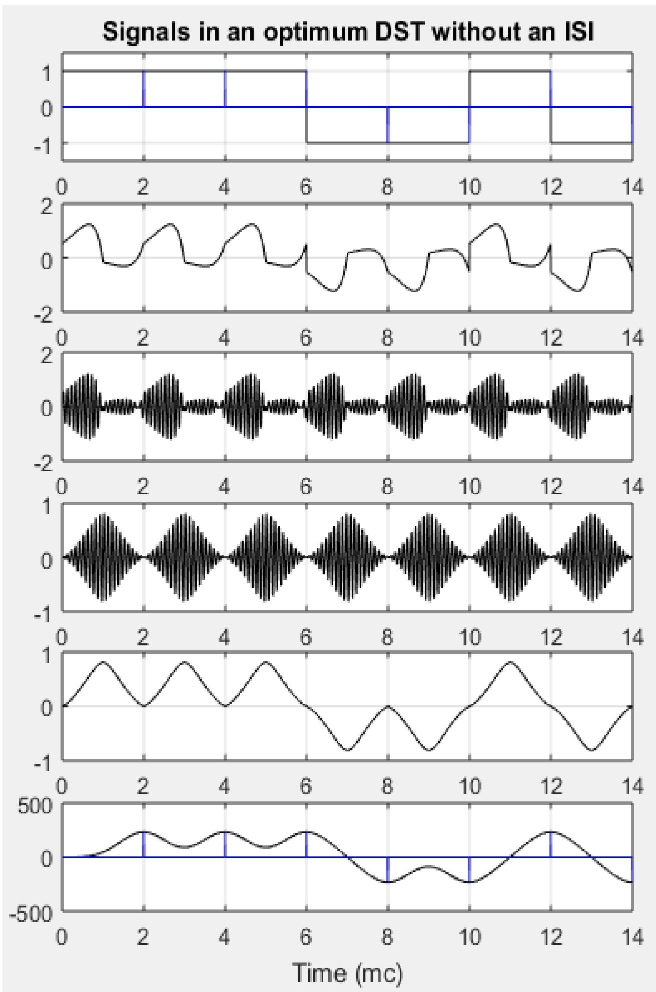

Fig. 8. Signals in various sections of the SPD as the result of the experiment

VII. Conclusion

1. A new structure of optimal reception of optimal finite signals without intersymbol interference is proposed for data transmission system with a linear bandlimited communication channel with multiple poles.

2. A comparative noise immunity analysis of this structure shows that the energy gain of the new optimal modem is 1.9 dB versus known one at the average error probability per bit equal to 10-4.

3. The developed method of optimal finite signals transmission and reception over a bandlimited communication channel can be applicable for new 5G technologies

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