Simulation of a Six-Degree Manipulator

Sergei Ivanov, Tatiana Zudilova
ITMO National Research University (ITMO University)
Saint Petersburg, Russian Federation
Serg_ie@mail.ru, Zudilova@ifmo.spb.ru

Andrei Mikalauskas, Irina Osetrova,
Lubov Ivanova
ITMO National Research University (ITMO University)
Saint Petersburg, Russian Federation
Mikalayskas@gmail.com, Irina@ifmo.spb.ru

***Abstract*—In paper develops a structure of a mechanical manipulator with six degrees of freedom. A mathematical model of the manipulator is investigated using the matrix method and the matrix Lagrange second-kind dynamic equations. As a result of the analysis of the mathematical model, generalized coordinates, velocities and accelerations for the links of the robot are obtained, the spatial trajectory of the movement of the gripper of the manipulator is determined. The developed mathematical model of the robot allows you to design automated robotic systems with six degrees of freedom.**

I. INTRODUCTION

Recently, automated robotic systems have been actively applied to the economic development of industrial countries. Such automated systems include industrial robots, manipulators and can achieve up to 75% increase in industrial production. Industrial robots are widely used in the automotive industry, in electronics, in metalworking, in the production of rubber and plastics, in the food industry, in pharmaceuticals, as well as in other fields.

Industrial robot - automatically controlled multi-purpose drive mechanism that performs industrial automation tasks. A manipulator is a machine, a mechanism that consists of consecutive connected links. Manipulators are designed to capture and move objects in three-dimensional space. The manipulator can be controlled by an operator or a programmable electronic controller. The two connected links of the manipulator represent a kinematic pair.

The task of developing a nonlinear mathematical model of a mechanical manipulator with six degrees of freedom is considered. When creating dynamic equations of motion, the matrix method and the Lagrange equations of the second kind are used. The mechanical manipulator under study is used to create automated systems for the maintenance of machinery and equipment. The mechanical manipulator performs functions similar to those of a human hand. In industrial production, technological manipulators are used to creating automated complexes. Manipulators are used for assembly operations, for machining, welding, for servicing mechanisms, devices, and machines, for removing and installing equipment, for changing parts, for tools.

The main function of the mechanical manipulator is determined by the kinematic scheme and consists of moving the gripper with the processing object to a given point in space. The paper proposes a kinematic structure of a mechanical manipulator with six degrees of freedom.

The mathematical model is represented by a nonlinear system of six ordinary second-order differential equations.

To build a solution to the proposed system, analytical methods are used. The purpose of the study of the model of the manipulator is to determine the main dynamic characteristics. The basic dynamic characteristics of the motion of a manipulator with six degrees of freedom are determined.

Currently, a large number of scientific articles devoted to the problems of simulation of robots. We give some scientific work in the field of research of manipulators. In the article [1], adaptive impedance control was developed for a robotic manipulator using neural networks and the Lyapunov method. Simulations are carried out to verify the proposed control.

In the article [2] discusses forging manipulators for the modern industry. Forging manipulator systems with a large payload is usually characterized by a large output load and a large capacitive load. The study analyzed the general kinematics and forces of the main mechanism of the manipulator.

In the article [3] discusses the use of flexible manipulators in various robotic applications. Research is being carried out in the field of modeling, sensor systems and controllers for the use of flexible robotic manipulators. A brief description of the main modeling methods is presented, followed by an overview of practical alternatives to sensor systems. A detailed review of control strategies for flexible manipulators is presented.

In the article [4], adaptive neural networks are used to design controls for suppressing vibrations of a flexible robotic manipulator. To improve the accuracy of the description of the elastic deflection of a flexible manipulator, the system is modeled using an approach with concentrated spring masses. Numerical simulation for a concentrated model of a flexible robotic system is carried out to test the effectiveness of control.

In the article [5] proposes a new adaptive controller that uses the Bat algorithm to control a robotic arm. The sliding mode controller is one of the control methods that provide high reliability and low tracking error.

In the article [6] presents a conveyor mechanism, which is a classic case of manipulating a conveyor with one degree of mobility. The geometry and kinematics of the conventional manipulator used in classical mechanics are considered. The manipulator is used on lifting platforms, on chairs for the disabled, on cranes, forklifts.

In the article [7], the neural network controller is designed to suppress the vibration of a flexible robotic manipulator system with an input dead zone. The flexible manipulator model is based on the concentrated spring-mass method. The dynamics of the manipulator and the influence of the input dead zone are approximated by a neural network with a radial basis function.

In the article [8] article presents the kinematic diagram of the manipulator for lifting and solves the nonlinear dynamic model of the lift using the analytical method of transformations.

The authors developed a computer program [9] for calculating models of industrial manipulators. The program is registered and implemented at several enterprises for the design and automation of control systems for industrial manipulators.

A computer program allows the calculation of any kinematic schemes of manipulators up to six degrees of freedom. The program consists of the following modules: for calculating the displacement matrices, for calculating the absolute coordinates of the robot grip, for determining the kinetic and potential energy of the manipulator links, for compiling the Lagrange matrix equations and for solving the dynamic equations of the manipulator by the analytical method of transformations.

II. MODELING OF THE MANIPULATOR WITH SIX DEGREES OF FREEDOM

Consider the kinematic scheme of the manipulator (Fig. 1), which includes five rotational kinematic pairs and one translational pair.

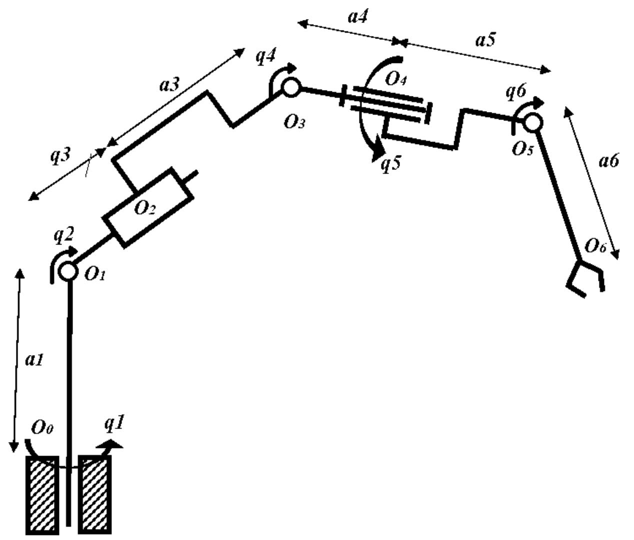

Fig. 1. Kinematic scheme of the manipulator

When developing the dynamic equations of a robot, we apply the matrix method and Lagrange matrix dynamic equations. In the matrix method, extended transition matrices from one coordinate system of the manipulator links to another coordinate system are used. The links of the industrial manipulator are modeled by rods, the joints are modeled by cylindrical joints and sliding joints without friction.

By the matrix method [10], we apply the transition matrices for the coordinate systems of the links of the manipulator. When moving from one coordinate system to the next coordinate system, no more than four movements are necessary: a turn around an axis, two shifts along the axes and a turn.

Define the coordinate system of the links of the robot in points $O_1, O_2, O_3, O_4, O_5, O_6$. The initial coordinate system is connected to the fixed base of the manipulator at O_0.

Let us take as a generalized q_i coordinates of a robot with six degrees of freedom: the angle of rotation around the rack, the angle of inclination of the rack, the length of the extension of the arm, the angle of rotation of the arm, the angle of rotation and inclination of the gripper. Here we measure angles in radians, lengths in centimeters.

The three-dimensional model of an industrial manipulator was built and modeling was carried out in a specialized computer program (Fig. 2).

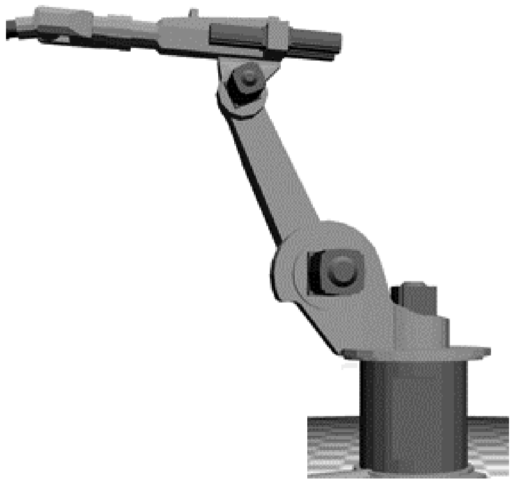

Fig. 2. Three-dimensional model of the manipulator

Transition from the coordinate system O_0 to the coordinate system O_1 performed by the following: rotation around the z axis at an angle q_1, a shift along the z axis by a_1 and rotate around the x axis at an angle $-\pi/2$.

Transition from the coordinate system O_1 to the coordinate system O_2 performed by the following: a shift along the z axis by q_3 and rotate around the x axis at an angle q_2.

Transition from the coordinate system O_2 to the coordinate system O_3 performed by the following: a shift along the z axis by a_3

Transition from the coordinate system O_3 to the coordinate system O_4 performed by the following: a shift along the z axis by a_4 and rotate around the x axis at an angle q_4.

Transition from the coordinate system O_4 to the coordinate system O_5 performed by the following: rotation around the z axis at an angle q_5, a shift along the z axis by a_5

Transition from the coordinate system O_5 to the coordinate system O_6 performed by the following: a shift along the z axis by a_6 and rotate around the x axis at an angle q_6.

We introduce the radius vector of points O_i, in the i – th coordinate system: $R_i = \left[x_i \quad y_i \quad z_i \quad 1\right]^T$.

Transition matrix $A_{i-1,i}$ connects the radius of the vector in the coordinate systems i – 1 and i: $R_{i-1} = A_{i-1,i} R_i$.

Denote function: $C_i = Cos\left[q_i\right], S_i = Sin\left[q_i\right]$.

The transition matrices to the next coordinate system are defined:

$$A_{01} = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_2 & -S_2 & 0 \\ 0 & S_2 & C_2 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{34} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_4 & -S_4 & 0 \\ 0 & S_4 & C_4 & a_4 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{45} = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & a_5 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{56} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_6 & -S_6 & 0 \\ 0 & S_6 & C_6 & a_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The transition matrices from the fixed coordinate system O_0 to O_i are determined: $A_{0i} = A_{01} A_{12} \ldots A_{i-1\ i}$

$$A_{02} = \begin{bmatrix} C_1 & -S_1 S_2 & -C_2 S_1 & -q_3 S_1 \\ S_1 & C_1 S_2 & C_1 C_2 & C_1 q_3 \\ 0 & -C_2 & S_2 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{03} = \begin{bmatrix} C_1 & -S_1 S_2 & -C_2 S_1 & -a_3 C_2 S_1 - q_3 S_1 \\ S_1 & C_1 S_2 & C_1 C_2 & a_3 C_1 C_2 + C_1 q_3 \\ 0 & -C_2 & S_2 & a_1 + a_3 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{04} = \begin{bmatrix} C_1 & -C_4 S_1 S_2 - C_2 S_1 S_4 & -C_2 C_4 S_1 + S_1 S_2 S_4 & -a_3 C_2 S_1 - a_4 C_2 S_1 - q_3 S_1 \\ S_1 & C_1 C_4 S_2 + C_1 C_2 S_4 & C_1 C_2 C_4 - C_1 S_2 S_4 & a_3 C_1 C_2 + a_4 C_1 C_2 + C_1 q_3 \\ 0 & -C_2 C_4 + S_2 S_4 & C_4 S_2 + C_2 S_4 & a_1 + a_3 S_2 + a_4 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{05} = \begin{bmatrix} C_1 C_5 + \left(-C_4 S_1 S_2 - C_2 S_1 S_4\right) S_5 & C_5\left(-C_4 S_1 S_2 - C_2 S_1 S_4\right) - C_1 S_5 & -C_2 C_4 S_1 + S_1 S_2 S_4 & -a_3 C_2 S_1 - a_4 C_2 S_1 - q_3 S_1 + a_5\left(-C_2 C_4 S_1 + S_1 S_2 S_4\right) \\ C_5 S_1 + \left(C_1 C_4 S_2 + C_1 C_2 S_4\right) S_5 & C_5\left(C_1 C_4 S_2 + C_1 C_2 S_4\right) - S_1 S_5 & C_1 C_2 C_4 - C_1 S_2 S_4 & a_3 C_1 C_2 + a_4 C_1 C_2 + C_1 q_3 + a_5\left(C_1 C_2 C_4 - C_1 S_2 S_4\right) \\ \left(-C_2 C_4 + S_2 S_4\right) S_5 & C_5\left(-C_2 C_4 + S_2 S_4\right) & C_4 S_2 + C_2 S_4 & a_1 + a_3 S_2 + a_4 S_2 + a_5\left(C_4 S_2 + C_2 S_4\right) \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Denote the coordinates $\left(x_6, y_6, z_6\right)$ of the gripper in the coordinate system O_6. In a fixed system O_0, the coordinates of the gripper are defined:

$$\begin{aligned} x_{06} = & -a_3 C_2 S_1 - a_4 C_2 S_1 - q_3 S_1 + \\ & a_5\left(-C_2 C_4 S_1 + S_1 S_2 S_4\right) + a_6\left(-C_2 C_4 S_1 + S_1 S_2 S_4\right) + \\ & \left(C_1 C_5 + \left(-C_4 S_1 S_2 - C_2 S_1 S_4\right) S_5\right) x_6 + \\ & (C_6\left(C_5\left(-C_4 S_1 S_2 - C_2 S_1 S_4\right) - C_1 S_5\right) + \\ & \left(-C_2 C_4 S_1 + S_1 S_2 S_4\right) S_6) y_6 + \\ & \left(C_6\left(-C_2 C_4 S_1 + S_1 S_2 S_4\right) - \left(C_5\left(-C_4 S_1 S_2 - C_2 S_1 S_4\right) - C_1 S_5\right) S_6\right) z_6, \end{aligned}$$

$$y_{06} = a_3C_1C_2 + a_4C_1C_2 + C_1q_3 +$$
$$a_5\left(C_1C_2C_4 - C_1S_2S_4\right) + a_6\left(C_1C_2C_4 - C_1S_2S_4\right) +$$
$$\left(C_5S_1 + \left(C_1C_4S_2 + C_1C_2S_4\right)S_5\right)x_6 +$$
$$(C_6\left(C_5\left(C_1C_4S_2 + C_1C_2S_4\right) - S_1S_5\right) +$$
$$\left(C_1C_2C_4 - C_1S_2S_4\right)S_6)y_6 +$$
$$\left(C_6\left(C_1C_2C_4 - C_1S_2S_4\right) - \left(C_5\left(C_1C_4S_2 + C_1C_2S_4\right) - S_1S_5\right)S_6\right)z_6,$$

$$z_{06} = a_1 + a_3S_2 + a_4S_2 + a_5\left(C_4S_2 + C_2S_4\right) +$$
$$a_6\left(C_4S_2 + C_2S_4\right) + \left(-C_2C_4 + S_2S_4\right)S_5x_6 +$$
$$(C_5C_6\left(-C_2C_4 + S_2S_4\right) + \left(C_4S_2 + C_2S_4\right)S_6)y_6 +$$
$$\left(C_6\left(C_4S_2 + C_2S_4\right) - C_5\left(-C_2C_4 + S_2S_4\right)S_6\right)z_6 .$$

Determine the kinetic energy of all links of the robot using transition matrices:

$$T_i = \frac{1}{2} tr\left(\dot{A}_{0i} H_i \dot{A}_{0i}^T\right) , \qquad (1)$$

Where H_i –link inertia matrix, m_i - link weight.

Determine the potential energy of the links:

$$P_i = -m_i G^T A_i R_i , \qquad (2)$$

where $R_i = \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}^T$ – matrix column coordinates of the center of gravity link,

$G_i^T = \begin{bmatrix} 0 & 0 & -g & 0 \end{bmatrix}$ – matrix line of gravitational acceleration.

Total potential energy is determined:

$$P = g\begin{pmatrix} a_1\left(m_1 + m_2 + m_3 + m_4 + m_5 + m_6\right) + \\ Sin[q_2 + q_4]\left(a_5m_5 + \left(a_5 + a_6\right)m_6\right) + \\ Sin[q_2]\left(a_4\left(m_4 + m_5 + m_6\right) + a_3\left(m_3 + m_4 + m_5 + m_6\right)\right) \end{pmatrix}$$

We write the system of dynamic equations of motion of the manipulator using the Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q'_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial P}{\partial q_i} - Q_i = 0 , \qquad (3)$$

where Q_i –generalized forces generated by the electric drive link.

Substituting the kinetic, potential energy and generalized forces in the Lagrange equations, we get the system of equations of motion of the manipulator with six degrees of freedom.

$$q_1^{''}(0.005m_1 + \left(0.005 + 0.5a_3^2\right)m_3 +$$
$$\left(0.005 + 0.5a_3^2 + a_3a_4 + 0.5a_4^2\right)m_4 + 0.5a_5^2)m_5 +$$
$$(0.005 + 0.5a_3^2 + a_3a_4 + 0.5a_4^2 +$$
$$\left(0.005 + 0.5a_3^2 + a_3a_4 + 0.5a_4^2 + 0.5a_5^2 + a_5a_6 + 0.5a_6^2\right)m_6 +$$
$$Sin\left[2q_2\right]Sin\left[2q_4\right](-0.5a_5^2m_5 + \left(-0.5a_5^2 - a_5a_6 - 0.5a_6^2\right)m_6) +$$
$$Cos\left[2q_2\right]Cos\left[2q_4\right]\left(0.5a_5^2m_5 + \left(0.5a_5^2 + a_5a_6 + 0.5a_6^2\right)m_6\right) +$$
$$Cos\left[2q_2\right]\begin{pmatrix} a_4^2\left(0.5m_4 + 0.5m_5 + 0.5m_6\right) + \\ a_3^2\left(0.5m_3 + 0.5m_4 + 0.5m_5 + 0.5m_6\right) + \\ a_3a_4\left(m_4 + m_5 + m_6\right) \end{pmatrix} +$$
$$Cos\left[q_2\right]\begin{pmatrix} a_4\left(2m_4 + 2m_5 + 2m_6\right) + \\ a_3\left(2m_3 + 2m_4 + 2m_5 + 2m_6\right) \end{pmatrix} q_3 +$$
$$\left(m_3 + m_4 + m_5 + m_6\right)q_3^2 +$$
$$2Cos\left[q_2\right]Cos\left[q_4\right]\left(a_6m_6 + a_5\left(m_5 + m_6\right)\right)\left(Cos\left[q_2\right]\left(a_3 + a_4\right) + q_3\right) -$$
$$2Sin\left[q_2\right]Sin\left[q_4\right]\left(a_6m_6 + a_5\left(m_5 + m_6\right)\right) +$$
$$\left(Cos\left[q_2\right]\left(a_3 + a_4\right) + q_3\right) + m_2\left(0.005 + q_3^2\right)) == Q_1 ,$$

$$q_2^{''}\begin{pmatrix} 0.005m_2 + \left(0.005 + a_3^2\right)m_3 + \left(0.005 + a_3^2 + 2a_3a_4 + a_4^2\right)m_4 + \\ \left(0.005 + a_3^2 + 2a_3a_4 + a_4^2 + a_5^2\right)m_5 \end{pmatrix} +$$
$$q_2^{''}\begin{pmatrix} \left(0.005 + a_3^2 + 2a_3a_4 + a_4^2 + a_5^2 + 2a_5a_6 + a_6^2\right)m_6 + \\ 2Cos\left[q_4\right]\left(a_3 + a_4\right)\left(a_6m_6 + a_5\left(m_5 + m_6\right)\right) \end{pmatrix} =$$
$$-gCos\left[q_2 + q_4\right]\left(a_6m_6 + a_5\left(m_5 + m_6\right)\right) +$$
$$Q_2 + Sin\left[2q_2\right]a_4^2\left(-0.5m_4 - 0.5m_5 - 0.5m_6\right)\left(q_1^{'}\right)^2 +$$
$$Sin\left[2q_2\right]a_3^2\left(-0.5m_3 - 0.5m_4 - 0.5m_5 - 0.5m_6\right)\left(q_1^{'}\right)^2 +$$
$$(Sin\left[2\left(q_2 + q_4\right)\right](-0.5a_5^2m_5 + (-0.5a_5^2 - a_5a_6 - 0.5a_6^2)m_6) +$$
$$Sin\left[q_2 + q_4\right]\left(a_5\left(-m_5 - m_6\right) - a_6m_6\right)q_3)\left(q_1^{'}\right)^2 +$$
$$a_3(-Cos\left[q_4\right]Sin\left[2q_2\right]\left(a_5m_5 + \left(a_5 + a_6\right)m_6\right)\left(q_1^{'}\right)^2 -$$
$$-m_3\left(gCos\left[q_2\right] + Sin\left[q_2\right]q_3\left(q_1^{'}\right)^2\right) -$$
$$\left(m_4 + m_5 + m_6\right)\left(gCos\left[q_2\right] + Sin\left[q_2\right]\left(2Cos\left[q_2\right]a_4 + q_3\right)\left(q_1^{'}\right)^2\right) -$$
$$Sin\left[q_4\right]\left(a_5m_5 + \left(a_5 + a_6\right)m_6\right)\left(Cos\left[2q_2\right]\left(q_1^{'}\right)^2 - 2q_2^{'}q_4^{'}\right)) +$$
$$a_4(-Cos\left[q_4\right]Sin\left[2q_2\right]\left(a_6m_6 + a_5\left(m_5 + m_6\right)\right)\left(q_1^{'}\right)^2 -$$

$$(m_4+m_5+m_6)\left(gCos[q_2]+Sin[q_2]q_3\left(q_1^{'}\right)^2\right)-$$
$$Sin[q_4]\left(a_6m_6+a_5(m_5+m_6)\right)\left(Cos[2q_2]\left(q_1^{'}\right)^2-2q_2^{'}q_4^{'}\right)),$$

$$q_3^{''}(m_3+m_4+m_5+m_6)=Q_3+(Cos[q_2]a_4(m_4+m_5+m_6)+$$
$$Cos[q_2]a_3(m_3+m_4+m_5+m_6)+$$
$$Cos[q_2+q_4]\left(a_5m_5+(a_5+a_6)m_6\right)+$$
$$(m_2+m_3+m_4+m_5+m_6)q_3)\left(q_1^{'}\right)^2,$$

$$q_4^{''}\left(0.005(m_4+m_5+m_6)+a_5^2m_5+a_5^2m_6+2a_5a_6m_6+a_6^2m_6\right)=$$
$$Q_4+Sin\left[2(q_2+q_4)\right]a_5^2(-0.5m_5-0.5m_6)\left(q_1^{'}\right)^2+$$
$$a_6m_6(-gCos[q_2+q_4]+$$
$$\begin{pmatrix}\left(-0.5Sin[q_4]-0.5Sin[2q_2+q_4]\right)(a_3+a_4)- \\ 0.5Sin\left[2(q_2+q_4)\right]a_6-Sin[q_2+q_4]q_3\end{pmatrix}\left(q_1^{'}\right)^2-$$
$$Sin[q_4](a_3+a_4)\left(q_2^{'}\right)^2)-a_5(gCos[q_2+q_4](m_5+m_6)+$$
$$(Sin\left[2(q_2+q_4)\right]a_6m_6+$$
$$\left(0.5Sin[q_4]+0.5Sin[2q_2+q_4]\right)a_3(m_5+m_6)$$
$$+\left(0.5Sin[q_4]+0.5Sin[2q_2+q_4]\right)a_4(m_5+m_6)+$$
$$Sin[q_2+q_4](m_5+m_6)q_3)\left(q_1^{'}\right)^2+$$
$$Sin[q_4](a_3+a_4)(m_5+m_6)\left(q_2^{'}\right)^2),$$

$$q_5^{''}(m_5+m_6)=200Q_5\ ,$$

$$q_6^{''}m_6=200Q_6\,. \tag{4}$$

The correctness of the mathematical model of the industrial manipulator is justified by the use of the universally recognized matrix method in the kinematics of manipulators, by the application of the traditional matrix Lagrange equations in the dynamics of manipulators.

The fifth and sixth equations of the system are easily integrated:

$$q_5(t)=\frac{100t^2Q_5}{m_5+m_6},$$

$$q_6(t)=\frac{100t^2Q_6}{m_6}\,.$$

To solve the remaining three differential equations of system (4), we apply the method of polynomial transformations [9,10] with the following parameters:

$$m_1=200\ ,m_2=60\ ,m_3=30\ ,m_4=20\ ,m_5=20\ ,m_6=20\ ,$$

$$Q_1=6000,Q_2=60000,Q_3=0.01,$$

$$Q_4=20000,Q_5=0.01,Q_6=0.01,$$

$$a_1=30\ ,a_2=20\ ,a_3=20\ ,a_4=30\ ,a_5=30\ ,a_6=20\ .$$

For industrial MP manipulators with the circuit in Fig. 1, the kinematic characteristics were calculated for technical parameters of masses and lengths of links of the industrial manipulator.

The transformation method [11] allows us to find a solution with all the nonlinear components of the original system. The method of transformation [12] allows us to construct a solution of a nonlinear system of differential equations in an analytical form.

The solution of the system of three differential equations (4) is obtained by the method of transformations:

$$q_1(t)=-0.151594Cos[0.6971t]+0.118335Cos[1.1278t]+$$
$$0.118849Sin[0.6971t]+0.0117864Sin[1.1278t],$$
$$q_2(t)=-0.191824Cos[0.6664t]+0.14823Cos[1.1331t]+$$
$$0.175084Sin[0.6664t]+0.0104015Sin[1.1331t],$$

$$q_3(t)=-0.286225Cos[0.0831t]+0.215533Cos[1.0636t]+$$
$$2.29318Sin[0.0831t]+0.0294979Sin[1.0636t],$$

$$q_4(t)=-0.163246Cos[0.1301t]+0.113956Cos[1.1341t]+$$
$$1.40722Sin[0.1301t]-0.0162264Sin[1.1341t],$$

$$q_5(t)=-0.100686Cos[0.0961t]+0.0700462Cos[1.141t]+$$
$$1.32824Sin[0.0961t]-0.0194242Sin[1.141t],$$

$$q_6(t)=-0.252572Cos[0.1992t]+0.0145165Cos[1.8115t]+$$
$$1.32734Sin[0.1992t]+0.108237Sin[1.8115t].$$

Verification of the industrial manipulator model is performed by parallel modeling in a specialized computer program.

Fig. 3 shows the calculation of the generalized coordinates for the manipulator.

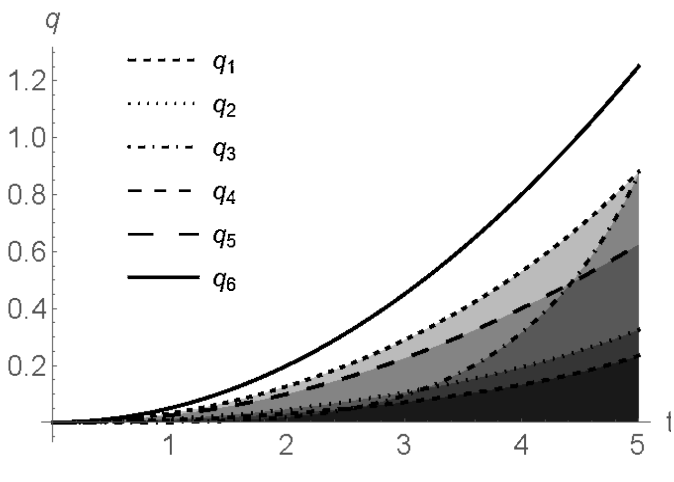

Fig. 3. Coordinates of links of the manipulator

Fig. 4 shows the calculation of the generalized velocities for the manipulator.

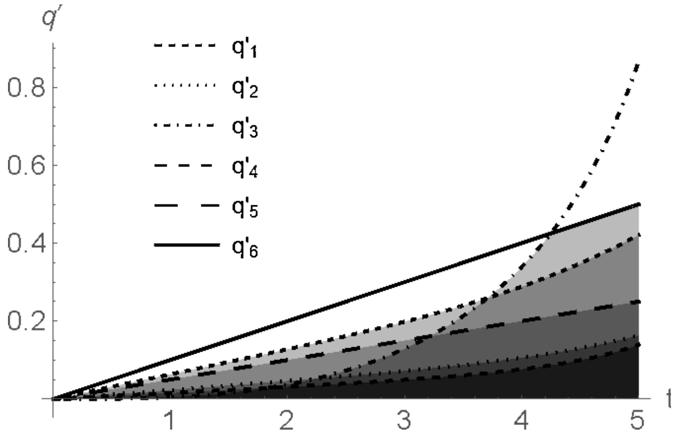

Fig. 4. Speed of links of the manipulator

Fig. 5 shows the calculation of the generalized accelerations for the manipulator.

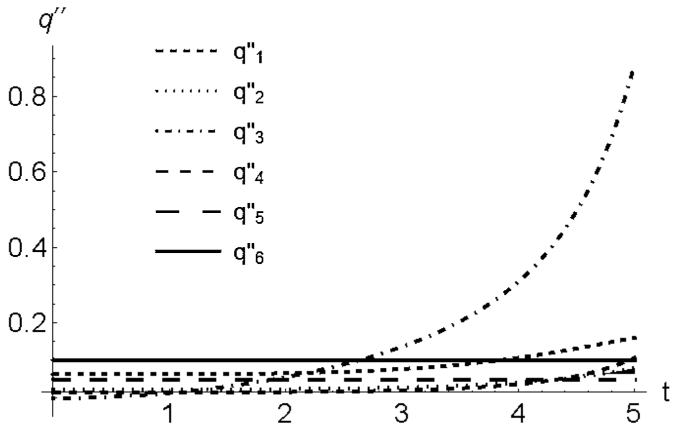

Fig. 5. Acceleration of the links of the manipulator

Fig. 6 shows the spatial trajectory of movement of the gripper arm relative to the fixed base of the rack.

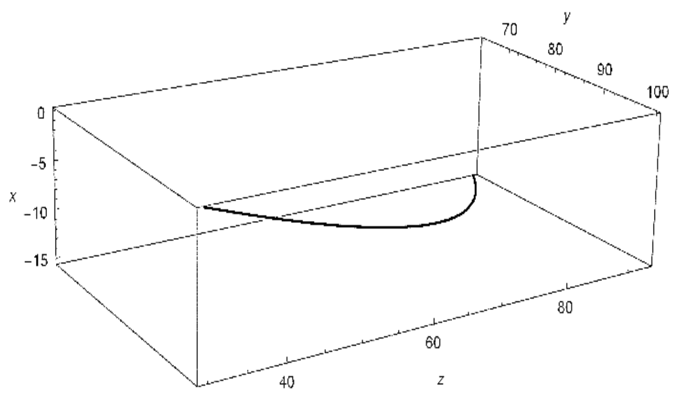

Fig. 6. The trajectory of the gripper arm

The resulting spatial trajectory of the robot grip coincides with the actual trajectory for industrial manipulators, which also confirms the reliability of the calculations of the manipulator model.

III. Conclusion

A kinematic scheme of a manipulator with six degrees of freedom has been developed and a mathematical model of a manipulator has been investigated using the matrix method. As a result of the analysis of the mathematical model, the coordinates, velocities, and accelerations of the links of the manipulator are determined. The work defines the spatial trajectory of the movement of the gripper arm in a fixed coordinate system. The developed kinematic scheme of the manipulator allows determining the dynamic characteristics of the manipulator. Mathematical calculations were checked in specialized computer mathematical packages. The mathematical model of the manipulator allows the design of manipulators with six degrees of freedom and the development of automated robotic systems. The authors plan to generalize this approach for the design of manipulators with many degrees of freedom.

References

[1] W. He, Y. Dong, C. Sun "Adaptive neural impedance control of a robotic manipulator with input saturation" *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 46.3, 2015, pp. 334-344.

[2] R. Aversa et al. "Kinematics and forces to a new model forging manipulator", *American Journal of Applied Sciences*, 14.1, 2017, pp. 60-80.

[3] C. T. Kiang, A. Spowage, C. K. Yoong "Review of control and sensor system of flexible manipulator", *Journal of Intelligent & Robotic Systems*, 77.1, 2015, pp. 187-213.

[4] C. Sun, W. He, J. Hong "Neural network control of a flexible robotic manipulator using the lumped spring-mass model", *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 47.8, 2016, pp. 1863-1874.

[5] M. Rahmani, A. Ghanbari, M.M. Ettefagh "Robust adaptive control of a bio-inspired robot manipulator using bat algorithm", *Expert Systems with Applications*, 56, 2016, pp. 164-176.

[6] R. V. Petrescu et al. "Geometry and determining the positions of a plan transporter manipulator", *Journal of Mechatronics and Robotics*, 1.2, 2017, pp. 118-126.

[7] W. He, Y. Ouyang, J. Hong "Vibration control of a flexible robotic manipulator in the presence of input deadzone", *IEEE Transactions on Industrial Informatics*, 13.1, 2016, pp. 48-59.

[8] T. V. Zudilova, S. E. Ivanov, L. N. Ivanova, "The automation of electromechanical lift for disabled people with control from a mobile device", *Computing Conference,* IEEE, 2017, pp. 668-674.

[9] S. E. Ivanov, "The program of analysis of dynamic models of polynomial structure for manipulators", *Certificate RF,* 2018, #2018619899

[10] E. I. Vorobyov, S. A. Popov, G. I. Sheveleva, *Mechanics of industrial robots: in 3 books, Part 1: Kinematics and dynamics,* M.: Higher. school, 1988, p. 304.

[11] G. I. Melnikov, S. E. Ivanov, V. G. Melnikov, "The modified Poincare-Dulac method in analysis of autooscillations of nonlinear mechanical systems", *Journal of Physics*: Conference Series, 570(2), IOP Publishing, 2014, p.022002.

[12] G. I. Melnikov, N. A. Dudarenko, K. S. Malykh, L. N. Ivanova, V. G. Melnikov, "Mathematical models of nonlinear oscillations of mechanical systems with several degrees of freedom", *Nonlinear Dynamics and Systems Theory*, 17(4), 2017, pp. 369-375.