A Hybrid Method for Constructing Optimal Motion Path for Robot Manipulators While Avoiding Obstacles

Sergei Ivanov, Lubov Ivanova, Andrei Mikalauskas
ITMO National Research University (ITMO University)
Saint Petersburg, Russian Federation
serg_ie@mail.ru, ln2305@yandex.ru, mikalayskas@gmail.com

***Abstract*—The scientific problem of constructing the optimal spatial trajectory of the gripper relative to the fixed base of the manipulator strut is considered taking into account the obstacle bypass. A new hybrid ABI method has been developed for constructing the optimal trajectory of the manipulator's grip taking into account obstacle avoidance. The ABI hybrid method has six main steps. The method combines a finite element mesh approach, a graph model construction method, A * method for finding the minimum path for a graph, a B-spline interpolation method, polynomial approximation, and a matrix method. The practical construction of the optimal trajectory of the grip of the manipulator is taking into account the avoidance of obstacles. A theorem is presented that determines the relationship between the length of time the manipulator grip moves in equally accelerated and equally slow sections. The theorem is used for the subsequent optimization of the manipulator travel time in sections.**

I. INTRODUCTION

Consider the task of constructing an optimal spatial trajectory of the movement of the grip relative to the fixed base of the arm of the manipulator, taking into account the circumvention of the obstacle.

The problem of determining the optimal path of the manipulator grip when bypassing the obstacle is solved. It is assumed that the movement of the manipulator grip can be divided into three sections: equidistant, uniform at a constant speed and equidistant. We believe at the beginning of time the capture is at rest. The total time to move the grip to the endpoint depends on the acceleration time to the operating speed, the time to move at a constant operating speed, and the stopping time. The limited operating speed of the manipulator is determined by the technical parameters of the manipulator and is assumed to be maximumly permissible. We assume that the main passage time of the manipulator is the time of uniform movement at a constant operating speed. Minimize the total length of the manipulator grip path and determine the fastest path - the optimal trajectory along which the manipulator grip moves at the maximum operating speed. The path is defined in the manipulator work area and it is assumed that the manipulator can perform any path in the work area. In the work area, there is an obstacle object which bypasses the manipulator grip to avoid the collision. The path of the manipulator seizure obstacle is constructed on the Stage assuming that the manipulator can perform any movement on the Stage.

It is assumed that the operating movement of the manipulator grip on the main section takes place at a constant speed and the obstacle bypass takes place on an even section of the grip motion. The assumption of uniform capture speed is necessary, for example, when the manipulator moves objects containing liquid fluid. Smoothing of movement path is necessary to satisfy uniform movement of manipulator grip at constant operating speed and to ensure continuity and smooth movement of grip.

Considering that the operating speed of the manipulator grip on the main uniform section of the movement is constant, the optimization of the trajectory leads to the reduction of time of the manipulator operating stroke.

II. RELATED WORKS

Many scientific papers are devoted to the methods of constructing trajectories for manipulators.

In works [1-11] methods of construction of manipulators movement paths are investigated.

In the study [1], higher-order inverse kinematics methods were proposed with time-optimal planning of the motion path for kinematically redundant manipulators. Kinematic redundancy is allowed and used in trajectory planning when solving higher-order inverse kinematics. Optimization results confirmed experimentally.

In [2], a nonlinear dynamic optimization method is presented for planning the trajectory of the spatial parallel Stuart-Hoff manipulator. The planning method consists of minimizing the power consumption of the actuator of the manipulator and minimizing the error when positioning the grip of the manipulator. A numerical solution procedure based on the finite element method is applied.

The study [3] presented a new approach for time-optimal planning of the trajectory of the redundant manipulator in three-dimensional workspaces. The proposed approach generates a trajectory for gripping the manipulator, taking into account both the kinematic limitations of the manipulator and the presence of obstacles. The task of optimizing the trajectory in time is solved using the genetic algorithm with multiple populations.

In [4], an optimal method for joint trajectory planning using the equations of direct kinematics of a freely floating space robot is presented. Bezier curves are used to describe joint paths. The differential evolution algorithm with the premature processing strategy is applied to find the optimal solution.

In [5], methods were proposed for solving the applied problem of planning a time-optimal path to prevent collisions of grinding manipulators. A method for planning a time-optimal trajectory between any two points is proposed based on the trajectory estimation mechanism using an annealing simulation algorithm. The annealing simulation algorithm generates new solutions based on the combined stochastic perturbation method.

The article [6] presents the methodology of synthetic optimal planning of the trajectory of robotic manipulators. The path is interpolated using a fifth-order B-spline and then optimized using a genetic algorithm. The fifth-order B-splines interpolation method allows you to limit the path in the kinematic limits of speed, acceleration, and jerk while satisfying jerk continuity.

In [7], the problem of designing partially balanced plane-symmetric parallel manipulators using optimal motion planning was considered. To solve the problem of balancing forces, redistribution of moving masses is applied, which reduces the variable dynamic loads for the manipulator. Balancing is accompanied by an increase in the mass of moving links, which negatively affects the torques. The study develops a balancing method without adding additional masses, based on minimizing the dynamic loads of the mechanical system of the manipulator by reducing the acceleration of its centre of mass.

In the article [8], a new trajectory planning algorithm is presented, based on switching the search strategy depending on the context for manipulators working in limited workspaces. The presented algorithm monitors the progress of the search and uses different search strategies in different parts of the search space. This allows you to solve the problems of planning the trajectory of the manipulator for very limited workspaces.

The article [9] considers the planning of the trajectory of an underwater excess manipulator in the presence of obstacles taking into account hydrodynamic effects. Trajectory planning is based on minimizing the energy needed to overcome hydrodynamic effects. The proposed method is used to plan the movement of a manipulator with three degrees of freedom, avoiding a point obstacle.

In the study [10], a trajectory planning algorithm was proposed for a manipulator with six degrees of freedom. The polynomial trajectory planning algorithm provides continuous angular acceleration and stable engine operation. Trajectory planning is carried out in Cartesian space using the spatial arc interpolation algorithm.

In [11], radial basis functions are used to create smooth trajectories of manipulator robots. Gaussian interpolation by radial basis functions is introduced taking into account the boundary conditions. The proposed approach is compared with classical trajectory planning methods based on polynomial and trigonometric models.

In the above works [1-11] various methods of planning the path of manipulators movement are presented under the corresponding assumptions and limitations. In the presented hybrid method, in the sixth step, the B-spline interpolation is applied similarly to operation [6] to improve the trajectory quality and compared with the interpolation of the Bezier curves [4]. Unlike the above-mentioned works, the hybrid method uses a whole set of methods and approaches to determine the optimal trajectory.

Let us introduce a new hybrid ABI method (Algorithm A * and B-spline Interpolation) for constructing the optimal trajectory of the manipulator's grip taking into account obstacle avoidance. The ABI hybrid method has six main steps. The method combines a finite element approach for constructing a mesh model taking into account an obstacle object, a graph model construction method, A * method for finding the minimum path for a graph, B-spline interpolation method, polynomial approximation, and matrix method.

III. Hybrid method ABI construction of trajectories grabbed concerning obstacle avoidance

Consider the ABI hybrid method for constructing an optimal manipulator grip trajectory taking into account obstacle avoidance.

Define the main stages of the hybrid ABI method.

The method uses the finite element mesh approach, the method of constructing a graph model, method A * of finding the minimum path for the graph, and the B-spline interpolation method.

At the first stage, we perform the triangulation of the grip area of the manipulator and construct a spatial grid model with elements in the form of a tetrahedron. In the mesh model, exclude the nodes that are located inside the obstacle object.

The Fig. 1 shows a spatial grid model in which an obstacle to walk around is represented by a cube.

For a manipulator spatial workspace that also contains an obstacle object or multiple obstacles, a uniform mesh model is constructed with the same finite-element geometric primitives in the form of a tetrahedron. We exclude the area with the obstacle object from the grid model. Reducing the pitch of a uniform grid increases the accuracy of the trajectory, the number of nodes, and also significantly increases the computational cost of calculating the model.

A cube is considered as an example of an obstacle object. The geometric shape of the obstacle object is not limited to the cube in question and may be of arbitrary shape.

In the second stage, we will build a three-dimensional graph model based on the grid model Fig. 2.

In the third stage, we apply the A * method for determining the shortest path in a graph model.

Fig. 3 shows the spatial trajectories constructed by the Dijkstra's algorithm (dashed line), Bellman-Ford algorithm, and the A * algorithm (black line). Algorithm A * uses a heuristic function - the Euclidean distance between nodes in three-dimensional space.

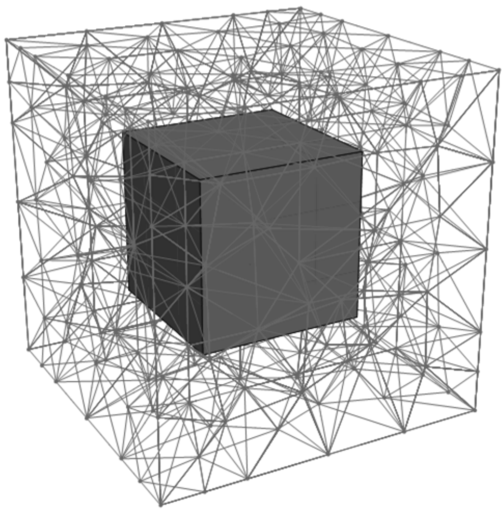

Fig. 1. Spatial mesh model with obstruction

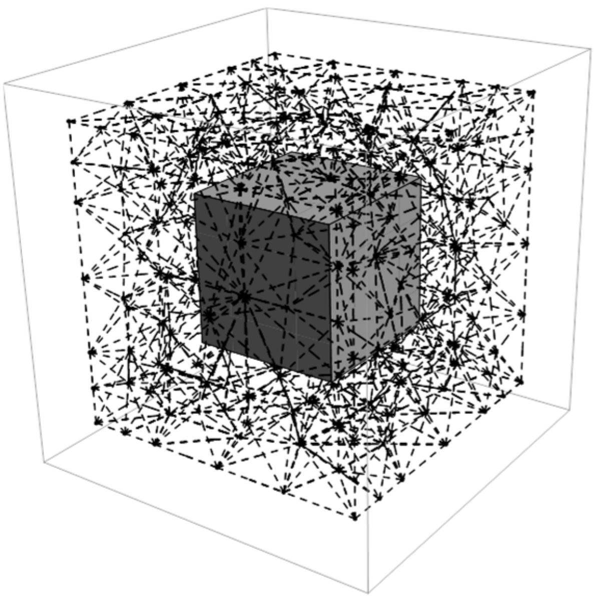

Fig. 2. 3D graph model

Dijkstra's algorithm works correctly only for graphs without edges with negative weight. The total running time of the algorithm is O(n^2+m) where n - is the number of vertices, m - is the number of edges.

The Bellman-Ford algorithm works correctly for graphs with negative edge weight.

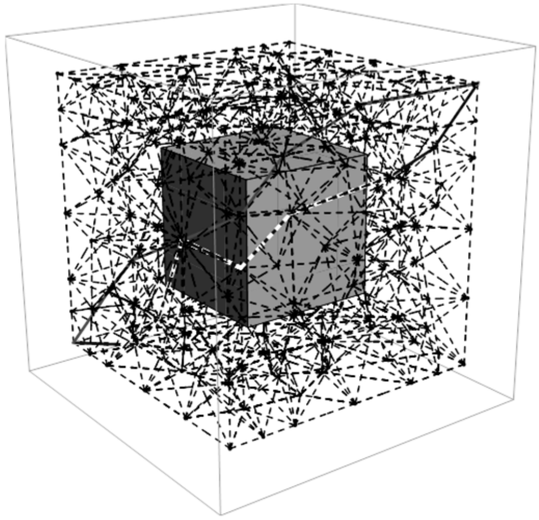

Fig. 3. Spatial trajectories of obstacle avoidance

The A * algorithm was developed in 1968 by P. E. Hart, N. J. Nilsson, B. Raphael based on Dijkstra's algorithm to increase productivity using a heuristic approach. As a heuristic function, we take the Euclidean distance between nodes in three-dimensional space. Algorithm A * step-by-step looks through all the paths from the start node to the end, until it finds the minimum one.

Note that applying different methods of finding the shortest path in the graph model represents different paths and total path length in connection with selecting a tetrahedron grid model with non-equal faces as the finite-element.

Table I shows the vertices, and total path length for trajectories constructed by the Dijkstra, Bellman-Ford algorithm, and the A * algorithm.

TABLE I. VERTICES, COORDINATES, AND TOTAL PATH LENGTH

ALGORITHM	VERTICES OF THE GRAPH	TOTAL PATH LENGTH
Dijkstra	{1, 140, 131, 94, 39, 206, 99, 46, 7}	13.695
Bellman - Ford	{1, 162, 130, 94, 134, 205, 25, 46, 7}	13.496
A*	{1, 140, 131, 94, 39, 206, 99, 145, 7}	13.295

Table I shows that for all three methods, the shortest path contains the same number of vertices.

Table I shows that the A * algorithm reduces the total path length by 0.4 compared to the Dijkstra algorithm and by 0.2 compared to the Bellman-Ford algorithm.

Fig. 4 shows the broken path of an obstacle bypass with a grip of a manipulator, constructed by means of algorithm A *.

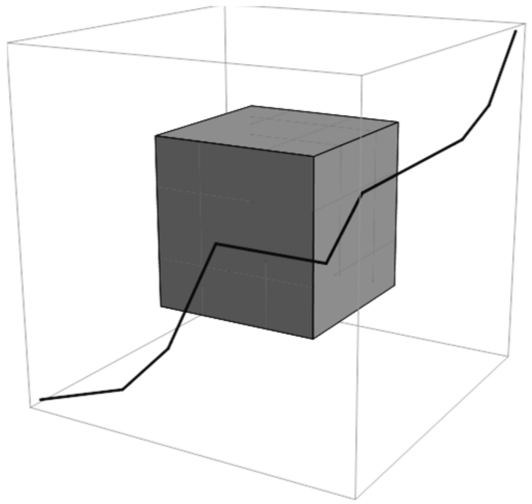

Fig. 4. Spatial broken path of obstacle avoidance.

For the considered problem of determining the optimal trajectory of the grip of the manipulator, the use of the algorithm A * reduces the total length of the grip path and, therefore, the travel time of the path.

At the fourth stage, for the broken path, we apply interpolation with third and fourth-order B-splines. Schoenberg introduced the concept of B-spline as an abbreviation for the base spline. B-spline interpolation differs from conventional spline interpolation in the definition of an auxiliary function for spline coefficients. When interpolating with B-splines, stitching is performed not at nodes, but control points. For an arbitrary curve, interpolation by cubic polynomials provides conjugation at the boundary points of the segments. Interpolation with B-splines guarantees the equality of the first and second derivatives when joining segments.

When interpolating polynomials of the order above the third, undulations appear. The interpolation by B-splines is characterized by a mismatch between the curve and the approximated points. If the number of nodes matches the degree of a spline, then the B-spline becomes a Bezier curve. Bezier curves were proposed in 1962 by Pierre Bezier of Renault for the design of car bodies. The Bezier curve is a special case of Bernstein polynomials. In the Bezier method, approximation by Bernstein polynomials is used. Bernstein's basis is a special case for basic B-spline functions.

Approximation by B-splines provides a more accurate approximation than approximation by Bernstein polynomials. For the Bezier method, the coordinates of each point of the curve are affected by all the vertices of the broken Bezier. To increase the order of the Bezier curve, it is necessary to increase the number of vertices of the broken Bezier.

To ensure that after the B-spline interpolation, the resulting trajectory does not enter the obstacle object, control points are selected during spline cross-linking, which are behind the obstacle by a given controlled distance.

Fig. 5 shows the interpolation for a broken path (grey line) of the gripper of the manipulator with B-splines of the third and fourth degrees (black line).

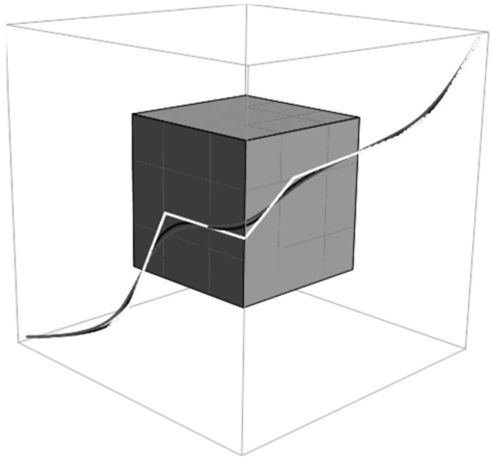

Fig. 5. Interpolations for the motion path of B-splines.

Fig. 6 shows the optimal spatial trajectory of the grip of the manipulator when avoiding an obstacle, constructed using B-spline interpolation.

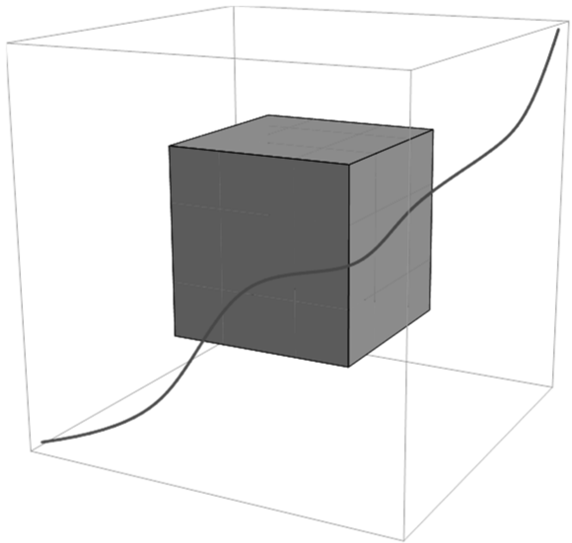

Fig. 6. Optimal spatial path of obstacle avoidance.

The length of the obstacle avoidance path constructed by interpolation by fourth-degree B-splines is 11.98.

In the fifth step, we apply the quadratic approximation of the trajectory constructed by interpolation with B-splines.

The approximation of the trajectory of avoiding the obstacle by polynomials of the second order is performed taking into account the boundary conditions, the exact coincidence of the starting and ending points of the trajectory. The parametric equations for the optimal trajectory of the grip of the manipulator of the form are defined:

$$Px(t) = 12.397t - 6.397t^2$$

$Py(t) = 1.145t + 4.855t^2$

$Pz(t) = 7.892t - 1.892t^2$

Fig. 7 shows the spatial trajectory of the grip of the manipulator when avoiding an obstacle, constructed according to the equations for $Px(t), Py(t), Pz(t)$.

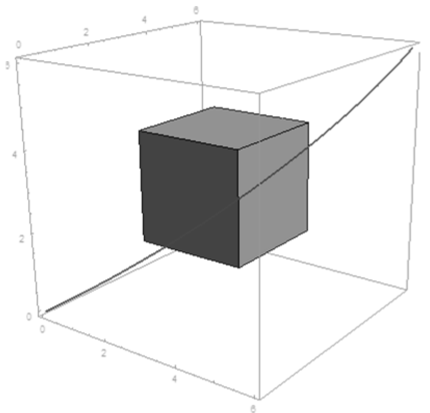

Fig. 7. The spatial trajectory of evading obstacles.

The length of the obstacle avoidance trajectory, constructed according to the equations, is 11.358.

The approximation of the trajectory by a polynomial of the second degree satisfies the equally variable motion of the grip of the manipulator. In this case, the manipulator's grip movement consists of three parts: uniformly accelerated, uniform and equally slow motion.

In the section of the uniformly accelerated movement, the grip of the manipulator is accelerated with constant positive acceleration from the initial state of the rest of the manipulator.

In the area of uniform motion, the grip of the manipulator moves at a constant working speed without acceleration.

In the area of equally slow motion, the manipulator grip is moved to the endpoint of the trajectory with constant negative acceleration. At the endpoint of the trajectory, the grip speed of the manipulator is zero.

For uniformly accelerated motion, the expressions for the velocities V, accelerations a, and the distance travelled S are valid:

$V_x = V_{x0} + a_x t,\ V_y = V_{y0} + a_y t, V_z = V_{z0} + a_z t,$

$a = const, a_x > 0, a_y > 0, a_z > 0,\quad V = \sqrt{V_x^2 + V_y^2 + V_z^2},$ $a = \sqrt{a_x^2 + a_y^2 + a_z^2},$

$S_x = V_{x0}t + \frac{a_x}{2}t^2, S_y = V_{y0}t + \frac{a_y}{2}t^2, S_z = V_{z0}t + \frac{a_z}{2}t^2,$

For uniform motion, the expressions for the velocities V, accelerations a and the distance travelled S are valid:

$V = const, a = 0, V = \sqrt{V_x^2 + V_y^2 + V_z^2},$

$S_x = V_{x0}t, S_y = V_{y0}t, S_z = V_{z0}t,$

For equally slow motion, the expressions for the velocities V, accelerations a, and the distance travelled S are valid:

$V_x = V_{x0} - a_x t,\ V_y = V_{y0} - a_y t, V_z = V_{z0} - a_z t,$

$a = const, a_x > 0, a_y > 0, a_z > 0,\quad V = \sqrt{V_x^2 + V_y^2 + V_z^2},$ $a = \sqrt{a_x^2 + a_y^2 + a_z^2},$

$S_x = V_{x0}t - \frac{a_x}{2}t^2, S_y = V_{y0}t - \frac{a_y}{2}t^2, S_z = V_{z0}t - \frac{a_z}{2}t^2,$

Fig. 8 shows the dependence $V_x(t)$ in the areas of

uniformly accelerated, uniform and equally slow motion of the manipulator grip.

Fig. 8. Sections of uniformly accelerated, uniform and equally slow motion of the manipulator grip.

Imagine a theorem that defines the relationship between the length of time the manipulator grip moves in equally accelerated and equally slow sections.

Theorem 1. When the manipulator moves with sections of uniformly accelerated, uniform and equally slow motion of the grip, the travel time of the equally accelerated and equally slow section of the path is directly proportional with a proportionality coefficient equal to the modulus of the ratio of total accelerations in these sections

Proof of the theorem.

Let us denote the transit time of the first uniformly accelerated section for t_1, the second uniform section for t_2, and the third equally slowed section for t_1.

Proportionality required to be proved $t_1 = t_3 \left|\frac{a_3}{a_1}\right|$

Here a_1 and a_3 are constant accelerations on an equally accelerated and equally slow section of the path.

Determine the full path by completing the addition of three sections.

$S_x = \frac{a_{x1}}{2}t_1^2 + V_{x2}t_2 + V_{x2}t_3 - \frac{a_{x3}}{2}t_3^2$

$S_y = \frac{a_{y1}}{2}t_1^2 + V_{y2}t_2 + V_{y2}t_3 - \frac{a_{y3}}{2}t_3^2$

$S_x = \frac{a_{z1}}{2}t_1^2 + V_{z2}t_2 + V_{z2}t_3 - \frac{a_{z3}}{2}t_3^2$

Fig. 9 shows that the full path is equal to the area of the trapezoid.

Define the area of the trapezoid:

$S_x = \frac{1}{2}(t_1 + 2t_2 + t_3)V_{x2},$

$S_y = \frac{1}{2}(t_1 + 2t_2 + t_3)V_{y2}$,

$S_z = \frac{1}{2}(t_1 + 2t_2 + t_3)V_{z2}$,

For the projection of speed on the second uniform section of motion, we write the equalities:

$V_{x2} = a_{x1}t_1$, $V_{y2} = a_{y1}t_1$, $V_{z2} = a_{z1}t_1$

Given the equalities, we equate the path to the areas of the trapezoid, we obtain a system of square algebraic equations:

$\frac{1}{2}a_{x1}t_1^2 + a_{x1}t_1t_2 + a_{x1}t_1t_3 - \frac{1}{2}a_{x3}t_3^2 =$

$\frac{1}{2}(t_1 + 2t_2 + t_3)a_{x1}t_1$,

$\frac{1}{2}a_{y1}t_1^2 + a_{y1}t_1t_2 + a_{y1}t_1t_3 - \frac{1}{2}a_{y3}t_3^2 =$

$\frac{1}{2}(t_1 + 2t_2 + t_3)a_{y1}t_1$,

$\frac{1}{2}a_{z1}t_1^2 + a_{z1}t_1t_2 + a_{z1}t_1t_3 - \frac{1}{2}a_{z3}t_3^2 =$

$\frac{1}{2}(t_1 + 2t_2 + t_3)a_{z1}t_1$,

Simplify the system

$\frac{1}{2}a_{x1}t_1t_3 - \frac{1}{2}a_{x3}t_3^2 = 0$,

$\frac{1}{2}a_{y1}t_1t_3 - \frac{1}{2}a_{y3}t_3^2 = 0$,

$\frac{1}{2}a_{z1}t_1t_3 - \frac{1}{2}a_{z3}t_3^2 = 0$,

Solving the system define

$a_{x1} = \frac{t_3}{t_1}a_{x3}$, $a_{y1} = \frac{t_3}{t_1}a_{y3}$, $a_{z1} = \frac{t_3}{t_1}a_{z3}$

Define the square of the full acceleration in the first section

$a_1^2 = a_{x1}^2 + a_{y1}^2 + a_{z1}^2 = \left(\frac{t_3}{t_1}a_{x3}\right)^2 + \left(\frac{t_3}{t_1}a_{y3}\right)^2 + \left(\frac{t_3}{t_1}a_{z3}\right)^2 = \left(\frac{t_3}{t_1}\right)^2(a_{x3}^2 + a_{y3}^2 + a_{z3}^2) = \left(\frac{t_3}{t_1}\right)^2 a_3^2$.

From here we get $t_1^2 = t_3^2\left(\frac{a_3}{a_1}\right)^2$

We take into account that time is a positive value.

Accordingly, $t_1 = t_3\left|\frac{a_3}{a_1}\right|$ and the theorem proved.

The theorem is used for the subsequent optimization of the manipulator travel time in sections.

At the sixth stage, we apply the matrix method [12] to determine the spatial coordinate functions of the manipulator grip.

We define the functions of generalized coordinates for a universal manipulator with six degrees of freedom, the kinematic diagram of which is shown in Fig. 9. In the manipulator diagram, there are five rotational kinematic pairs and one translational pair.

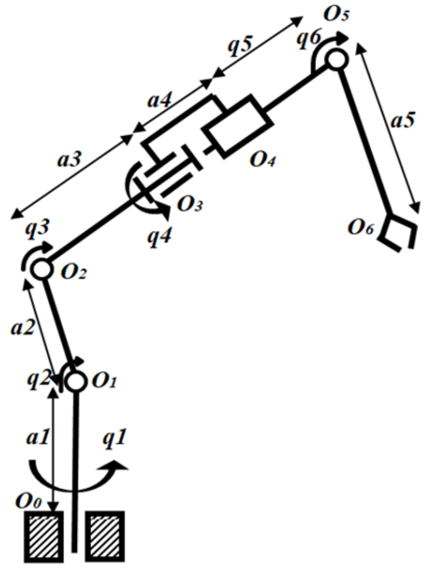

Fig. 9. The kinematic diagram of the manipulator.

The links of the industrial manipulator are modelled by rods, the joints are modelled by cylindrical joints and sliding joints. We assume that the friction in the joints is small and is not taken into account when deriving the robot model.

Define the coordinate system of the robot links at points $O_1, O_2, O_3, O_4, O_5, O_6$. The absolute coordinate system is connected with the fixed base of the manipulator at a point O_0.

We take as the generalized coordinates of the manipulator with six degrees of freedom the angles of rotation of the links q_1, q_2, q_3, q_4, q_6 and arm extension length q_5. Here we measure angles in radians, lengths in centimetres.

The matrix method determines the coordinates of the grip of the manipulator in the absolute coordinate system O_0 as a function of the generalized coordinates of the manipulator:

$x_6 = \mathrm{Sin}[q_1](\mathrm{Sin}[q_2]a_3 + \mathrm{Sin}[q_2 + q_3](a_4 + a_5 + q_5))$

$y_6 = -\mathrm{Cos}[q_1](\mathrm{Sin}[q_2]a_3 + \mathrm{Sin}[q_2 + q_3](a_4 + a_5 + q_5))$

$z_6 = a_1 + a_2 + \mathrm{Cos}[q_2]a_3 + \mathrm{Cos}[q_2 + q_3](a_4 + a_5 + q_5)$

To move the manipulator grip along an optimal trajectory, taking into account the obstacle bypass, we equate the coordinates of the manipulator grip to the equations for the optimal trajectory.

We obtain a system of three equations for determining $q_1(t), q_2(t), q_3(t)$:

$\mathrm{Sin}[q_1]\big(\mathrm{Sin}[q_2]a_3 + \mathrm{Sin}[q_2 + q_3](a_4 + a_5 + q_5)\big) = Px(t)$,

$\mathrm{Cos}[q_1]\big(-\mathrm{Sin}[q_2]a_3 - \mathrm{Sin}[q_2 + q_3](a_4 + a_5 + q_5)\big) = Py(t)$,

$a_1 + a_2 + \mathrm{Cos}[q_2]a_3 + \mathrm{Cos}[q_2 + q_3](a_4 + a_5 + q_5) = Pz(t)$

An analytical solution of the system of equations is obtained.

$q_1(t) = \frac{1}{2}\mathrm{ArcCos}[\frac{Py(t)^2 - Px(t)^2}{Px(t)^2 + Py(t)^2}]$,

$q_2(t) = \mathrm{ArcTan}[\frac{a_3^3 P_{\mathrm{za}} + a_3 P_{\mathrm{za}}(-a_6^2 + P_{\mathrm{xy}}^2 + P_{\mathrm{za}}^2)}{a_3^2(P_{\mathrm{xy}}^2 + P_{\mathrm{za}}^2)} -]$

$[\frac{\sqrt{-a_3^2 P_{\mathrm{xy}}^2\left(a_3^4 + (-a_6^2 + P_{\mathrm{xy}}^2 + P_{\mathrm{za}}^2)^2 - 2a_3^2(a_6^2 + P_{\mathrm{xy}}^2 + P_{\mathrm{za}}^2)\right)}}{a_3^2(P_{\mathrm{xy}}^2 + P_{\mathrm{za}}^2)}]$

$q_3(t) = -\mathrm{ArcTan}[\frac{1}{a_3^2 a_6(P_{\mathrm{xy}}^2 + P_{\mathrm{za}}^2)^2}(a_3^5(P_{\mathrm{xy}}^2 - P_{\mathrm{za}}^2) + a_3 a_6^2(a_6^2 - P_{\mathrm{xy}}^2 - P_{\mathrm{za}}^2)(P_{\mathrm{xy}}^2 - P_{\mathrm{za}}^2) - a_3^3(P_{\mathrm{xy}}^2 - P_{\mathrm{za}}^2)(2a_6^2 + P_{\mathrm{xy}}^2 + P_{\mathrm{za}}^2) + 2(a_3^2 - a_6^2)$

$P_{\mathrm{za}}\sqrt{-a_3^2 P_{\mathrm{xy}}^2\left(a_3^4 + \left(-a_6^2 + P_{\mathrm{xy}}^2 + P_{\mathrm{za}}^2\right)^2 - 2a_3^2\left(a_6^2 + P_{\mathrm{xy}}^2 + P_{\mathrm{za}}^2\right)\right)})]$

$a_6 = a_4 + a_5 + \frac{t^2 Q_5}{i_5^2 m_5 + i_6^2 m_6}$, $P_{\mathrm{za}} = Pz(t) - a_1 - a_2$,

$P_{\mathrm{xy}} = Px(t)^2 + Py(t)^2$

Analytical solution for $q_4(t), q_5(t), q_6(t)$ obtained from the dynamic matrix equations of Lagrange [13-15].

$q_4(t) = \frac{t^2 Q_4}{i_4^2 m_4 + i_5^2 m_5 + i_6^2 m_6}, q_5(t) = \frac{t^2 Q_5}{i_5^2 m_5 + i_6^2 m_6}, q_6(t) = \frac{t^2 Q_6}{i_6^2 m_6}.$

Here Q_k are the generalized forces created by the link drives, m_k is the mass of the link, i_k is the radius of inertia of the link.

Thus, the functions of the generalized coordinates of the gripper of the manipulator are determined for the movement of the gripper along the optimal trajectory with bypassing the obstacle.

The presented ABI hybrid method allows you to build the optimal trajectory of the manipulator when avoiding obstacles and to control the movement along the trajectory.

IV. Conclusion

In this paper, we consider the urgent task of constructing an optimal spatial trajectory of the grip movement relative to the fixed base of the manipulator strut, taking into account the obstacle bypass.

As a result of the hybrid method, the optimal path of manipulator grip motion is obtained, which ensures continuity and smoothness of grip motion. Minimizing the length of the path while bypassing the obstacle reduces the operating stroke time of the manipulator. Unlike the works considered, the hybrid method is multi-step and combines finite-element grid model construction methods, graph shortest path determination methods, interpolation, approximation, and matrix methods. The combination of these methods allows solving the problem of determining the optimal trajectory when bypassing the obstacle by the manipulator capture in the assumption of uniform movement of the capture on the main working section.

A theorem is presented that determines the relationship between the length of time the manipulator grip moves in equally accelerated and equally slow sections. The theorem is used for the subsequent optimization of the manipulator travel time in sections.

A new hybrid ABI method for constructing the optimal trajectory of the manipulator's grip taking into account obstacle avoidance is introduced. The ABI hybrid method has six main steps. The method combines a finite element mesh approach, a graph model construction method, A * method for finding the minimum path for a graph, a B-spline interpolation method, polynomial approximation, and a matrix method. The combination of grid, graph, interpolation and matrix methods allows you to completely solve the problem.

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