Logic Graphs for ALC, SHIF and SHOIN Description Logics

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Abstract—**In this article, we aim to develop a complete and convenient ontology visualization method, named "Logic graphs". The method is intended to represent the semantics of ontological structures, formulated as description logic axioms, and to be easy to learn, due to using existing visualization methods from mathematical theories, such as graph theory and Ch. S. Pierce's Existential graphs. The proposed system is sufficient to describe the description logics SHIF, and SHOIN, corresponding to OWL-Lite and OWL-DL languages. The method is exemplified with visualization of axioms from the DoCO ontology.**

I. Introduction

In recent years, ontologies have become common on the World-Wide Web. Ontologies are formal representations of concepts and complex relationships among them [1], [2]. They are used for sharing a common understanding of the structure of information among people or software agents. Ontologies visualization is helpful for making them easy to understand by casual users, for analyzing its structure on the high level and other tasks.

According to [3][4], there are visualization tools such as OWLViz, OntoTrack, KC-Viz, OntoRama, OntoGraf, FlexViz, TGViz, OLSVis, NavigOWL, SOVA, GrOWL, Ontodia, Graffoo, Graphol, and VOWL. Systems, like VOWL [5] or Graphol [6], that support almost all OWL language constructs. However, these systems have common drawback. They visualize ontological structures just labeling them, not representing their semantics. For example, compare the visualization of conjunction from Graphol, Fig. 1, with the corresponding Venn diagram [7], [8], Fig. 2. Venn diagram represents by itself that these two sets have common elements, while in Graphol the user has to know that hexagon denotes conjunction.

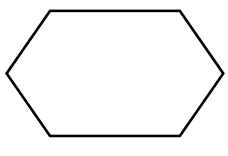

Fig.1.Conjunction in Graphol

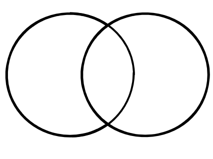

Fig.2. Conjunction in Venn diagrams

Therefore, our goal is to develop a complete and convenient ontology visualization method, named "Logic graphs", with two features. First, it should represent the semantics of the ontological structures, formulated as description logic axioms. Second, it should apply existing graphic primitives from mathematical theories, where it is possible.

The outline of the paper is as follows: Section 2 describes the classification of OWL sub-languages and corresponding description logics. Section 3 presents the Logic graph system in detail with examples from DoCO ontology, before the paper is concluded in Section 4.

II. OWL sub-languages and Description logics

Ontologies are denoted on the OWL language [9]. The Web Ontology Language OWL is a semantic markup language for publishing and sharing ontologies on the World Wide Web. The OWL 1 standard provided three increasingly expressive sub-languages: OWL Lite, OWL DL, and OWL Full. OWL Lite supports those users primarily needing a classification hierarchy and simple constraints. OWL DL provides the maximum expressiveness, retaining computational completeness (all conclusions are guaranteed to be computed) and decidability (all computations will finish in finite time). OWL DL includes all OWL language constructs, but they can be used only under certain restrictions.

The formal foundation of OWL is description logics. Description Logics (DLs) [10], [11], [12] are a family of logic languages, which can be used to represent the terminological knowledge of an application domain. The ALC (Attributive Language with complement) is a minimal DL of practical interest. Other DLs of this family are extensions of ALC. In this paper, we consider the SHIF and SHOIN languages as well, since they correspond to the OWL Lite and OWL DL sub-languages, respectively. The syntax and semantics of these logics is represented in Table I, where I denotes the interpretation function, Δ denotes the domain.

III. Logic graphs

We suggest applying existing graphic primitives from mathematical theories, such as Venn diagrams, Ch. S. Pierce's Existential graphs [13], [14], [15], [16], [17] and graph theory [18], where it is possible, in order to make our visualization method easy to learn, as the user won't have to learn new ones. As a result, we propose a complete and convenient ontology visualization method, named "Logic graphs", aimed to represent the semantics of description logic axioms.

In our previous work [19], we adopted the existential graph visualization method for description logic formulas. As the result, we developed the visualization method for ALC description logic. In this paper, we extend our visualization system for SHIF and SHOIN description logics. The resulting visualization system is presented in Table 2, where C, $C1$, and $C2$ are concepts, R – role and a and b – objects. Domain – a technical concept, introduced to represent domain concepts of roles.

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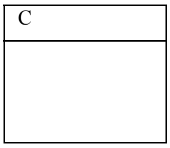
TABLE I. DESCRIPTION LOGICS

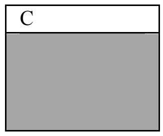
Logic	Name	Syntax	Semantic
ALC	concept	C	$C^I \subseteq \Delta^I$
	role	R	$R^I \subseteq \Delta^I \times \Delta^I$
	negation	$\neg C$	$\Delta^I \setminus C^I$
	conjunction	$C \sqcap D$	$C^I \cap D^I$
	disjunction	$C \sqcup D$	$C^I \cup D^I$
	existential restriction	$\exists R.\top$	$(\exists R.\top)^I = \{a \in \Delta^I: \exists b(a,b) \in R^I\}$
	universal restriction	$\forall R.C$	$(\forall R.C)^I = \{a \in \Delta^I: \forall b(a,b) \in R^I \to b \in C^I\}$
SHIF	inverse role	R^-	$(R^-)^I = \{(b,a) \in \Delta^I \times \Delta^I: (a,b) \in R^I\}$
	transitive role	R^+	$(R^+)^I = \cup_i (R^I)_i$, where $(a,b) \in (R^I)_i \wedge (b,c) \in (R^I)_i \to (a,c) \in (R^I)_i$
	functional role	$\leq 1R$	$\leq 1R \Leftrightarrow \{(a,b),(a,c)\} \subseteq R^I \Rightarrow b = c$
	role inclusion	$R \sqsubseteq S$	$R^I \subseteq S^I$
SHOIN	nominal	$\{o\}$	$\{o^I\}$
	number restrictions	$\geq nP$ $\leq nP$	$\{x \mid \#\{y. < x > y \in P^I\} \geq n$ $\{x \mid \#\{y. < x > y \in P^I\} \leq n$

TABLE II. LOGIC GRAPHS

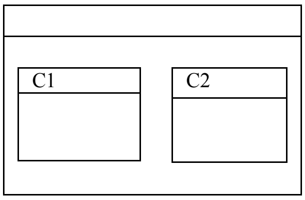
1. C



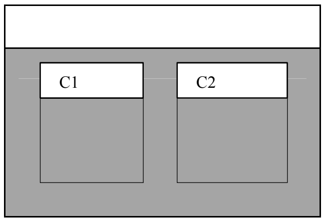
2. $\neg C$



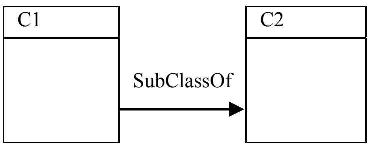
3. $C1 \sqcap C2$



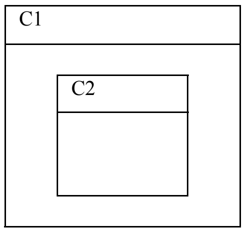
4. $C1 \sqcup C2$



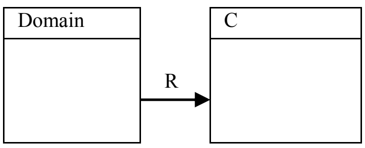
5. $C1 \sqsubseteq C2$



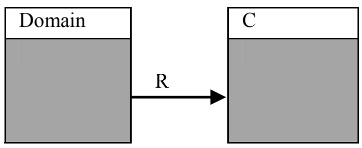
6. $C1 \equiv C2$

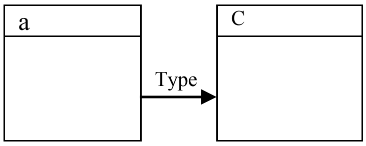
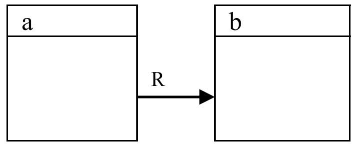
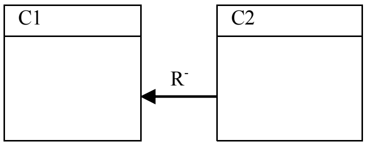
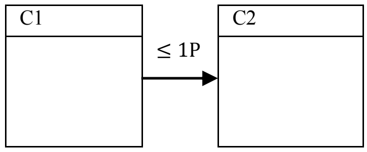
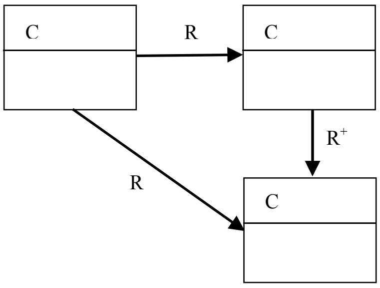
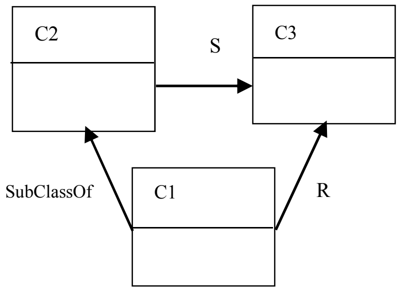
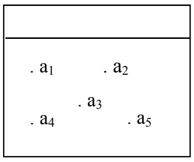
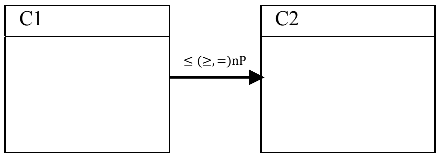


7. $\exists R.C$



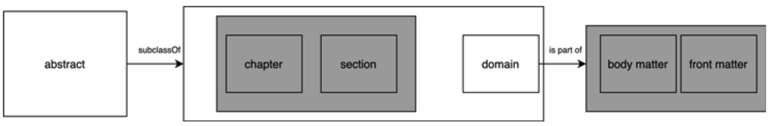
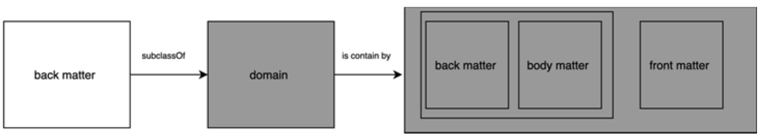
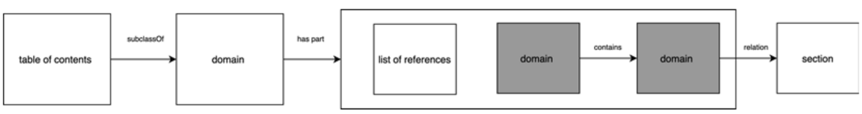
8. $\forall R.C$



9.	$a: C$		10.	$(a, b): R$	
11.	R-		12.	$\leq 1P$	
13.	R+		14.	$R \sqsubseteq S$	
15.	$\{a_1, \ldots, a_5\}$		16.	$\leq (\geq, =) nP$	

The Document Components Ontology (DoCO) [20] is an ontology that provides a structured vocabulary for document components. We use the DoCO ontology as an example for visualization, as it is a real ontology, used in different applications and it contains nontrivial axioms. We visualized some axioms using the developed method. See Table 3.

TABLE III. LOGIC GRAPHS FOR DOCO ONTOLOGY

	abstract $\sqsubseteq$ (chapter $\sqcup$ section) $\sqcap$ ($\exists$ *is part of* . *body matter* $\sqcup$ front matter)
	back matter $\sqsubseteq$ $\forall$*is contained by*.$\neg$(*back matter* $\sqcup$ body matter $\sqcup$ front matter)
	table of contents $\sqsubseteq$ $\exists$ *has part* . *list of references* $\sqcap$ ($\forall$ *contains*. $\exists$ *relation* . *section*)

IV. CONCLUSION

The developed visualization method, named “Logic graphs”, allows visualizing the semantics of logical expressions almost without new graphic primitives, except for being used in different mathematical theories, such as set theory, graph theory, and existential graphs. Logic graphs are sufficient for visualizing SHIF and SHOIN description logics that correspond to OWL Lite and OWL DL ontology languages.

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