The Algorithms for Processing of Imprecise Temporal Data

Aleksandr V. Flegontov, Vladimir V. Fomin, Sergey V. Maltsev
Herzen State Pedagogical University of Russia
St.Petersburg, Russia
flegontoff@yandex.ru, v_v_fomin@mail.ru, maltsevsergey@live.com

***Abstract*—In this paper we initiated the process of analyzing and addressing the problems related to storing and retrieval of inaccurate time information by means of building a new model of fuzzy temporal domain and corresponding data types, as well as methods for processing of fuzzy temporal relations between time points and/or intervals with the implementation of appropriate SQL procedures and functions.**

I. INTRODUCTION

The research caused by a practical need of improving the effectiveness of data representation methods, algorithms of data retrieval, procedures of unified processing of fuzzy temporal information, their prediction and interpretation during extraction of inaccurate-time information. Theoretical achievements and importance of research in the field of fuzzy temporal processing reflected in the following scientific and technical publications: designing database applications integrated with different time measurements [1], processing of fuzzy information in databases [2], [3], fuzzy time modelling in temporal relational databases [4], [5]. Applied fuzzy temporal data processing systems are needed in various areas as discussed in [6] and their implementation such as possibilistic valid-time model described in [7], [8], [9] is one of the objectives of industrial information technology. The need in usage of temporal data storage model actualized with appearance of temporal support defined in [10] in SQL:2011 standard. With appearance of the standard, developers of analytical systems have begun actively add temporal support to their products. Such implementation approach is being actively integrated by enterprises to meet the challenges associated with time.

II. PROCESSING OF POSSIBILISTIC TEMPORAL INFORMATION

A. Possibilistic temporal points and invervals

In order to implement the model that considers and handles temporal information, which contains inaccuracy or uncertainty, the notions of possibilistic time point and interval should be defined.

Possibilistic time point (pt) – approximate time stamp or time point on a time axis, which value is inaccurately defined or partially or completely unknown.

The following types represent the possibilistic time point:

1) *Time point value defined with a possible deviation*

Possibilistic value of time point's membership degree is set to 1, the value of possibilistic deviation from the time of occurrence is in the range of [0, 1]:

$$[pt, pt - \alpha, pt + \beta],$$

where $pt \in$ T – possibilistic time point, approximate time moment on the T axis; α and β – values of possibilistic deviation from the time of occurrence (Fig. 1).

Note, that all subsequent figures in this work represent examples of visualization and comparison calculations of temporal elements. On abscissa – a time axis T, ordinate – a degree of membership of elements to fuzzy set.

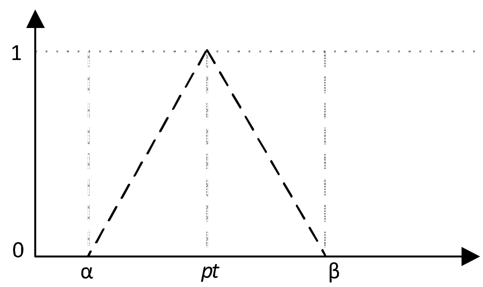

Fig. 1. Time point value defined with a possible deviation

2) *Time point value is not defined in the range*

Values in the range have the same possibility degree of occurrence = *PExtent*:

$$[\{pt_x, pt_y\}, PExtent],$$

where $\{pt_x, pt_y\} \in$ T – range of possibilistic time point values on the T axis. pt_x and pt_y can hold an empty value. In this case, the range of values takes the following form (Fig. 2).

$$[\{pt_x, \infty\}, PExtent] \vee [\{\infty, pt_y\}, PExtent]$$

3) *Time point value is not defined*

All values have the same possibility degree of occurrence = 0:

[undefined, 0]

Possibilistic interval ([pt_s, pt_e])) – period of time between two possibilistic time points.

For example, the possibilitic interval *[pt_1, pt_{e2}]* might look as shown on Fig. 3.

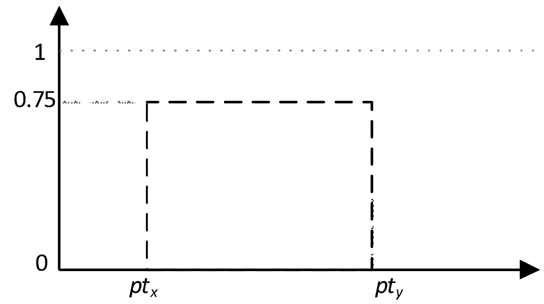

Fig. 2. Time point value is not defined in the range

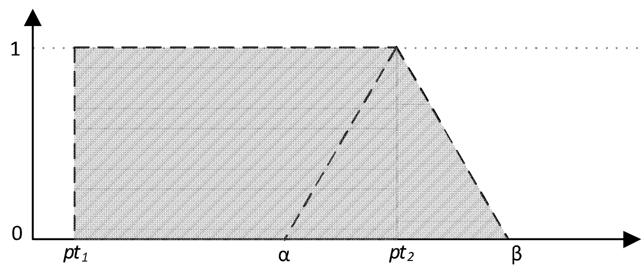

Fig. 3. Possibilistic interval

B. *Possibilistic relations between time points and intervals*

One of the most common models of time relations representation is Allen's interval algebra [12]. James F. Allen defined a set of 13 base relations that may exist between two time intervals (Table I). Allen's Algebra is restricted by the usage of relations between time periods (slots, intervals) and characterized by the measure of the duration of time intervals. However, the practice of temporal relations based not only on time intervals, but also on concept of time points. Time point and time interval are an integral part of the implementation of temporal data analysis.

To build the model that considers possibilistic relations between time points and/or intervals, the Allen's temporal algebra was expanded by decomposition of interval relations to relations between, first, two time points, and second, points and intervals. The introduced relations reflected in Table II.

On the basis of classical Allen's relations extensions defined in [13], [14], new introduced types of possibilistic time point and extended temporal relations between time points and/or intervals, the algebra of possibilistic relations between time points and/or intervals was developed as described below.

Possibilistic relation "Before"

Consider the snippet of algebra based on expansion of relation *Before* (Table I and Table II) up to three possibilistic types (Table III).

TABLE I. ALLEN'S TEMPORAL ALGEBRA

Relation name	Intervals I and J		Visualization
Before	I before J	$E_i < S_j$	I J t
After	I after J	$S_i > E_j$	J I t
Equal	I equals J	$S_i=S_j \wedge E_i=E_j$	J I t
Meets	I meets J	$E_i = S_j$	I J t
Met by	I met by J	$S_i = E_j$	J I t
Overlaps	I overlaps J	$S_i<S_j \wedge E_i<E_j \wedge E_i>S_j$	I J
Overlapped by	I overlapped by J	$S_i >S_j \wedge E_i>E_j \wedge S_i<E_j$	J I t
During	I during J	$S_i >S_j \wedge E_i<E_j$	I J t
Contains	I contains J	$S_i <S_j \wedge E_i>E_j$	J I t
Starts	I starts J	$S_i=S_j \wedge E_i<E_j$	I J t
Started by	I started by J	$S_i =S_j \wedge E_i>E_j$	J I t
Finishes	I finishes J	$S_i>S_j \wedge E_i=E_j$	J I t
Finished by	I finished by J	$S_i<S_j \wedge E_i=E_j$	I J t

TABLE II. INTRODUCED RELATIONS BETWEEN TIME POINTS AND INTERVALS

Relation name	Points A, B and Interval J		Visualization
Before	A before J A before B	$A_t < S_j$ $A_t < B_t$	A J B t
After	A after J A after B	$A_t > E_j$ $A_t > B_t$	B J A t
Equal	A equals B	$A_t = B_t$	A B t
Contains	J contains A	$S_j < A_t < E_j$	A J t
Starts	A starts J	$S_j = A_t < E_j$	A J t
Finishes	A finishes J	$S_j < A_t = E_j$	J A t

TABLE III. POSSIBILISTIC RELATION BEFORE

Relation type	Points a, b. Intervals IPeriod, JPeriod	View
Time points	*a* before *b*	*PosBefore (a, b[, PExtent_IN])*
Intervals	*IPeriod* before *JPeriod*	*PosBefore (IPeriod, JPeriod[, PExtent_IN])*
Time point and Interval	*a* before *IPeriod*	*PosBefore (a, IPeriod.startdate)*

To account the uncertainty effect the membership function of fuzzy set theory is applied, which allows quantifying the grade of membership of elements of fundamental set to the fuzzy set. The basis measures of the membership degree are real numbers in the interval from 0 to 1 introduced by Lotfi

Zadeh in [15]. A value of 0 means that the item is not included in the fuzzy set, 1 describes fully included item. Values between 0 and 1 characterize fuzzy included elements.

The relation designed to determine an occurrence of possibilistic time point "a" before "b". It takes the following form:

PosBefore (a, b[, PExtent_IN])

where a and b – mandatory compared values of time points, *PExtent_IN* $\in [0,1]$ (optional) – degree, required to fulfill the condition. The relation returns (*PExtent_OUT[, boolean]),* where *PExtent_OUT* $\in [0,1]$ – the resulting possibilistic degree of fulfillment, *boolean*: *true*, if *PExtent_OUT* ≥ *PExtent_IN; false*, if *PExtent_OUT<PExtent_IN.*

Let's apply possibilistic relation *Before* to input possibilistic time points:

1) Crisp value of time point "a"

1.1) Crisp value of time point "b"

$$PosBefore = \begin{cases} 0, & b \le a \\ 1, & b > a \end{cases}$$

1.2) The value of time point "b" defined with a possible deviation

$$PosBefore = \begin{cases} 0, & a \ge b\_\beta \\ 1 - \frac{a-b\_\alpha}{b-b\_\alpha}/2, & b\_\alpha \le a < b \\ \left(1 - \frac{a-b}{b\_\beta-b}\right)/2, & b \le a < b\_\beta \\ 1, & a < b\_\alpha \end{cases}$$

Possibilistic time point "b" with periods of a possible deviation of occurrence b_α and b_β (Fig. 4).

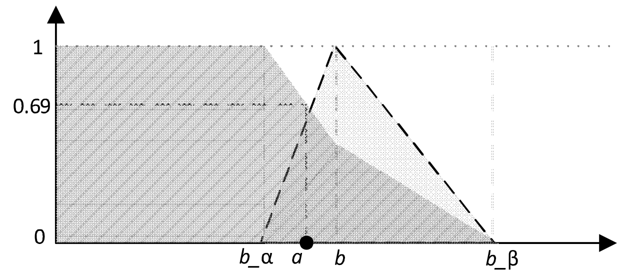

Fig. 4. Relation *Before*: the value of time point "b" defined with a possible deviation

1.3) The value of time point "b" is not defined in the range

Possibilistic time point "b" with possibilistic range of occurrence b_α, b_β, and possible degree *b_PExtent_IN* (Fig. 5).

$$PosBefore = \begin{cases} 0 * b\_PExtent\_IN, & b\_\beta \le a \\ \left(1 - \frac{a-b\_\alpha}{b\_\beta-b\_\alpha}\right) * b\_PExtent\_IN, & b\_\alpha \le a < b\_\beta \\ 1 * b\_PExtent\_IN, & a < b\_\alpha \end{cases}$$

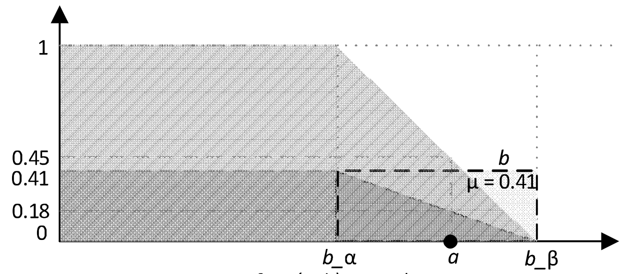

Fig. 5. Relation *Before*: the value of time point "b" is not defined in the range

In cases when b_α and/or b_β take empty values, for accurate calculation the start date and end date are placed with default values (*d_sd* and *d_ed*), which shall be determined in accordance with the subject area.

When b_α = *null* and b_β <> *null*:

$$PosBefore = \begin{cases} 0, & b\_\beta \le a \\ \left(1 - \frac{a-d\_sd}{b\_\beta-d\_sd}\right) * b\_PExtent\_IN, & d\_sd \le a < b\_\beta \\ 1 * b\_PExtent\_IN, & a < d\_sd \end{cases}$$

When b_α <> *null* and b_β = *null*:

$$PosBefore = \begin{cases} 1 * b\_PExt\_IN, & a \le b\_\alpha \\ \left(1 - \frac{a-b\_\alpha}{d\_ed-b\_\alpha}\right) * b\_PExt\_IN, & b\_\alpha \le a < d\_ed \\ 0 * b\_PExt\_IN, & d\_ed < a \end{cases}$$

When b_α = *null* and b_β = null:

$$PosBefore = \begin{cases} 0, & d\_ed \le a \\ \left(1 - \frac{a-d\_sd}{d\_ed-d\_sd}\right) * b\_PExtent\_IN, & d\_sd \le a < d\_ed \\ 1 * b\_PExtent\_IN, & a < d\_sd \end{cases}$$

1.4) The value of time point "b" is not defined

Possibilistic time point "b" with unknown time of occurrence.

The relation returns the original value of *PExtent_IN*, which is equal to 0.

2) The value of time point "a" defined with a possible deviation

2.1) Crisp value of time point "b"

$$PosBefore = \begin{cases} 0, & b \le a\_\alpha \\ 1 - \frac{a\_\beta-b}{a\_\beta-a}/2, & a < b \le a\_\beta \\ \left(1 - \frac{a-b}{a-a\_\alpha}\right)/2, & a\_\alpha < b \le a \\ 1, & b > a\_\beta \end{cases}$$

Possibilistic time point "a" with periods of a possible deviation of the occurrence α and β (Fig. 6).

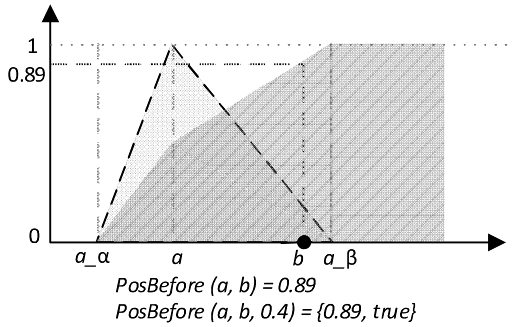

Fig. 6. Relation *Before*: crisp value of time point "*b*"

2.2) The value of time point "b" defined with a possible deviation

Possibilistic time point "*a*" with periods of a possible deviation of the occurrence a_α and a_β. Possibilistic time point "*b*" with periods of a possible deviation of the occurrence b_α and b_β (Fig. 7).

For each $a\_x \in A$, where $A = \{a, a\_\alpha, a\_\beta\}$, performed the following calculation of *pos_extent* degree:

$$pos\_extent = \begin{cases} 0, & a\_x > b\_\beta \\ 1 - \frac{a\_x - b\_\alpha}{b - b\_\alpha}/2, & b\_\alpha < a\_x \le b \\ \left(1 - \frac{a\_x - b}{b\_\beta - b}\right)/2, & b < a\_x \le b\_\beta \\ 1, & a\_x \le b\_\alpha \end{cases}$$

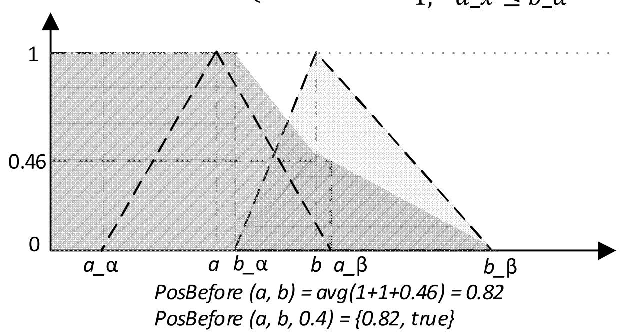

Fig. 7. Relation *Before*: the value of time point "b" defined with a possible deviation

The degree of possible occurrence "*a*" before "*b*" calculated using the average of *pos_extent* values:

PosBefore =AVG (pos_extent (a), pos_extent (a_α), pos_extent (a_β))

2.3) The value of time point "b" is not defined in the range

Possibilistic time point "*a*" with periods of a possible deviation of the occurrence a_α and a_β. Possibilistic time point "b" with possibilistic range of occurrence b_α, b_β, and possible degree *b_PExtent_IN* (Fig. 8).

For each $a\_x \in A$, where $A = \{a, a\_\alpha, a\_\beta\}$, performed the following calculation of *pos_extent* degree:

$$pos\_extent = \begin{cases} 0, & a\_x > b\_\beta \\ \left(1 - \frac{a\_x - b\_\alpha}{b\_\beta - b\_\alpha}\right) * b\_PExtent\_IN, & b\_\alpha < a\_x \le b\_\beta \\ 1 * b\_PExtent\_IN, & a\_x \le b\_\alpha \end{cases}$$

The degree of possible occurrence "*a*" before "*b*" calculated using the average of *pos_extent* values:

PosBefore =AVG (pos_extent (a), pos_extent (a_α), pos_extent (a_β))

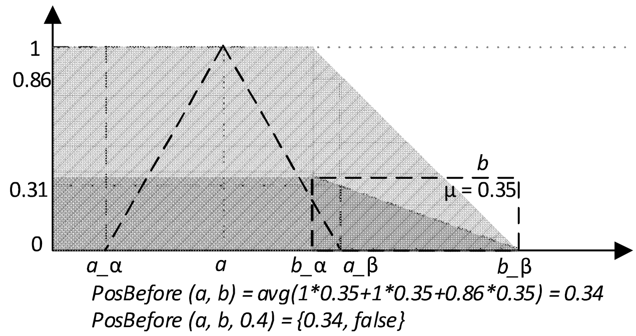

Fig. 8. Relation *Before*: the value of time point "*b*" is not defined in the range

2.4) The value of time point "b" is not defined

Possibilistic time point "*b*" with unknown time of occurrence.

The relation returns the original value of *PExtent_IN*, which is equal to 0.

3) The value of time point "a" is not defined in the range

3.1) Crisp value of time point "b"

Possibilistic time point "*a*" with possibilistic range of occurrence a_α, a_β, and possible degree *a_PExtent_IN* (Fig. 9).

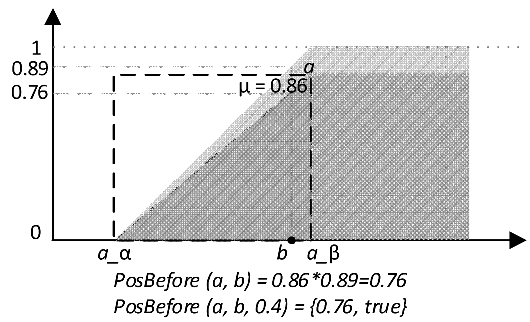

Fig. 9. Relation *Before*: crisp value of time point "*b*"

$$PosBefore = \begin{cases} 0, & a\_\alpha > b \\ \left(\frac{b - a\_\alpha}{a\_\beta - a\_\alpha}\right) * a\_PExt\_IN, & a\_\alpha \le b < a\_\beta \\ 1 * a\_PExt\_IN, & a\_\beta \le b \end{cases}$$

3.2) The value of time point "b" defined with a possible deviation

Possibilistic time point "*a*" with possibilistic range of occurrence a_α, a_β, and possible degree *a_PExtent_IN*. Possibilistic time point "*b*" with periods of a possible deviation of the occurrence b_α and b_β (Fig. 10).

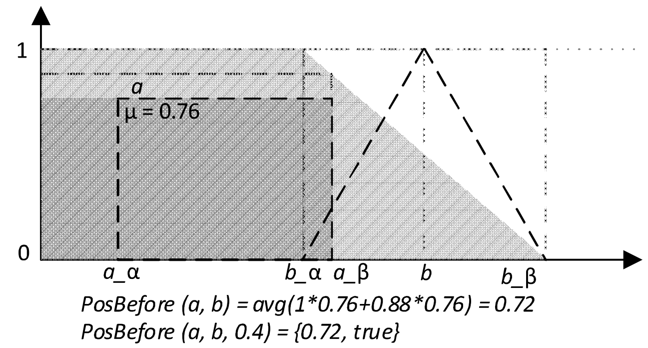

Fig. 10. Relation *Before*: the value of time point "b" defined with a possible deviation

For each $a\_x \in A$, where $A = \{a\_\alpha, a\_\beta\}$, performed the following calculation of *pos_extent* degree:

$$pos\_extent = \begin{cases} 0, & a\_x \geq b\_\beta \\ 1 - \frac{a\_x - b\_\alpha}{b - b\_\alpha}/2 * a\_PExtent\_IN, & b\_\alpha \leq a\_x < b \\ \left(1 - \frac{a\_x - b}{b\_\beta - b}\right)/2 * a\_PExtent\_IN, & b \leq a\_x < b\_\beta \\ 1 * a\_PExtent\_IN, & a\_x < b\_\alpha \end{cases}$$

The degree of possible occurrence "*a*" before "*b*" calculated using the average of *pos_extent* values*:*

PosBefore =AVG (pos_extent (a_α), pos_extent (a_β))

3.3) The value of time point "b" is not defined in the range

Possibilistic time point "*a*" with possibilistic range of occurrence *a_α*, a_β, and possible degree *a_PExtent_IN.* Possibilistic time point "*b*" with possibilistic range of occurrence *b_α*, b_β, and possible degree *b_PExtent_IN* (Fig. 11).

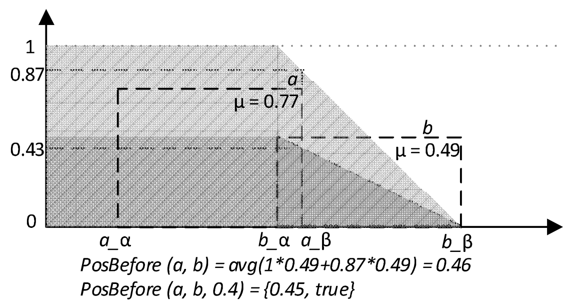

Fig. 11. Relation *Before*: the value of time point "*b*" is not defined in the range

For each $a\_x \in A$, where $A = \{a\_\alpha, a\_\beta\}$, performed the following calculation of *pos_extent* degree:

$$pos\_extent =$$

$$= \begin{cases} 1 * \min(a\_PE\_IN, b\_PE\_IN), & a\_x < b\_\alpha \\ \left(1 - \frac{a\_x - b\_\alpha}{b\_\beta - b\_\alpha}\right) * \min(a\_PE\_IN, b\_PE\_IN), & b\_\alpha \leq a\_x < b\_\beta \\ 0, & b\_\beta \leq a\_x \end{cases}$$

The degree of possible occurrence "*a*" before "*b*" calculated using the average of *pos_extent* value*:*

PosBefore =AVG (pos_extent (a_α), pos_extent (a_β))

3.4) The value of time point "b" is not defined

Possibilistic time point "*b*" with unknown time of occurrence.

The relation returns the original value of *PExtent_IN*, which is equal to 0.

Let's apply possibilistic relation *Before* to input possibilistic intervals:

PosBefore (IPeriod, JPeriod[, PExtent_IN]),

where IPeriod and *JPeriod* – mandatory compared interval values, *PExtent_IN∈ [0,1]* (optional) – degree, required to fulfill the condition. The relation returns (*PExtent_OUT[, boolean]),* where *PExtent_OUT ∈ [0,1]* – the resulting possibilistic degree of fulfillment, *boolean*: *true*, if *PExtent_OUT ≥ PExtent_IN; false*, if *PExtent_OUT<PExtent_IN.*

In accordance with classical crisp relation *Before* (Table I), possibillistic condition for the relation will look as follows:

PosBefore (IPeriod.enddate, JPeriod.startdate)

The relation can also determine an occurrence of possibilistic time point before interval as follows:

PosBefore (a, IPeriod[, PExtent_IN])

In accordance with classical crisp relation *Before* (Table II), possibillistic condition for the relation will look as follows:

PosBefore (a, IPeriod.startdate)

Possibilistic relation "Equals"

Consider expansion of another relation *Equals* (Table I and Table II) up to three possibilistic types (Table IV).

TABLE IV. POSSIBILISTIC RELATION EQUALS

Relation type	Points a, b. Intervals IPeriod, JPeriod	View
Time points	*a* equals *b*	*PosEqual (a, b[, PExtent_IN])*
Intervals	*IPeriod* equals *JPeriod*	*PosEqual (IPeriod, JPeriod[, PExtent_IN])*

The relation designed to determine an equality of possibilistic time points "*a*" and "*b*". It takes the following form:

PosEqual (a, b[, PExtent_IN])

where *a* and *b* – mandatory compared values of time points, *PExtent_IN ∈ [0,1]* (optional) – degree, required to fulfill the condition. The relation returns (*PExtent_OUT[, boolean]),* where *PExtent_OUT ∈ [0,1]* – the resulting possibilistic degree of fulfillment, *boolean*: *true*, if *PExtent_OUT ≥ PExtent_IN; false*, if *PExtent_OUT<PExtent_IN.*

Let's apply possibilistic relation *Equal* to input possibilistic time points:

1) Crisp value of time point "a"

1.1) Crisp value of time point "b"

$$PosEqual = \begin{cases} 0, & a <> b \\ 1, & a = b \end{cases}$$

1.2) The value of time point "b" defined with a possible deviation

$$PosEqual = \begin{cases} 0, & a \leq b\_\alpha \vee a > b\_\beta \\ \frac{a - b\_\alpha}{b - b\_\alpha}, & b\_\alpha < a \leq b \\ \frac{b\_\beta - a}{b\_\beta - b}, & b < a \leq b\_\beta \end{cases}$$

Possibilistic time point "b" with periods of a possible deviation of occurrence b_α and b_β (Fig. 12).

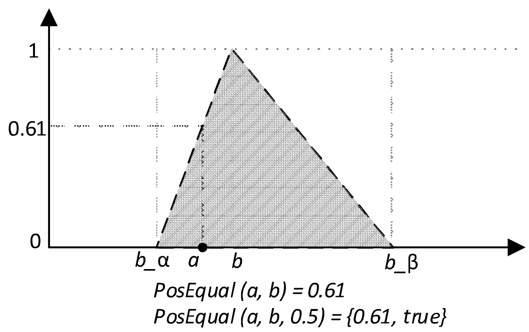

Fig. 12. Relation *Equals*: the value of time point "b" defined with a possible deviation

1.3) The value of time point "b" is not defined in the range

Possibilistic time point "b" with possibilistic range of occurrence b_α, b_β, and possible degree *b_PExtent_IN* (Fig. 13).

$$PosEqual = \begin{cases} 0, & a < b\_\alpha \vee a > b\_\beta \\ 1 * b\_PExtent\_IN, & b\_\alpha \leq a \leq b\_\beta \end{cases}$$

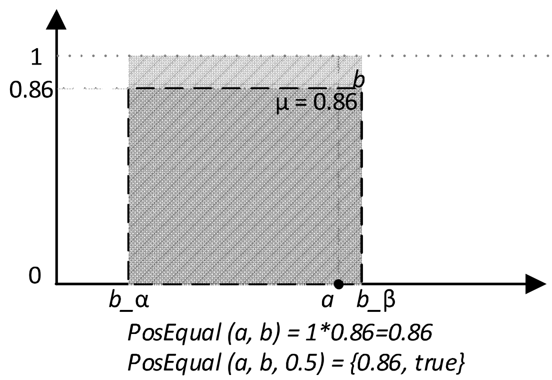

Fig. 13. Relation *Equals*: the value of time point "b" is not defined in the range

1.4) The value of time point "b" is not defined

Possibilistic time point "b" with unknown time of occurrence.

The relation returns the original value of *PExtent_IN*, which is equal to 0.

2) The value of time point "a" defined with a possible deviation

2.1) Crisp value of time point "b"

$$PosEqual = \begin{cases} 0, & b \leq a\_\alpha \vee b > a\_\beta \\ \frac{b - a\_\alpha}{a - a\_\alpha}, & a\_\alpha < b \leq a \\ \frac{a\_\beta - b}{a\_\beta - a}, & a < b \leq a\_\beta \end{cases}$$

Possibilistic time point "a" with periods of a possible deviation of the occurrence a_α and a_β (Fig. 14).

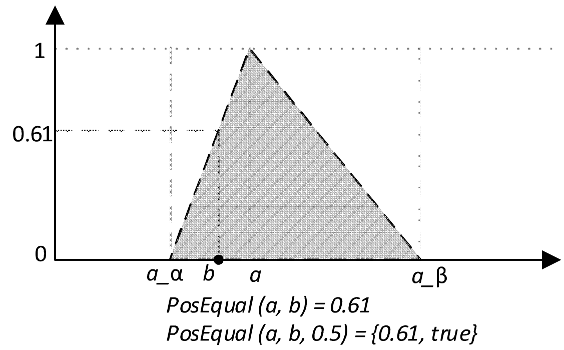

Fig. 14. Relation *Equals*: crisp value of time point "b"

2.2) The value of time point "b" defined with a possible deviation

Possibilistic time point "a" with periods of a possible deviation of the occurrence a_α and a_β. Possibilistic time point "b" with periods of a possible deviation of the occurrence b_α and b_β (Fig. 15).

For each $a\_x \in A$, where $A = \{a, a\_\alpha, a\_\beta\}$, performed the following calculation of *pos_extent* degree:

$$pos\_extent = \begin{cases} 0, & a\_x \leq b\_\alpha \vee a\_x > b\_\beta \\ \frac{a\_x - b\_\alpha}{b - b\_\alpha}, & b\_\alpha < a\_x \leq b \\ \frac{b\_\beta - a\_x}{b\_\beta - b}, & b < a\_x \leq b\_\beta \end{cases}$$

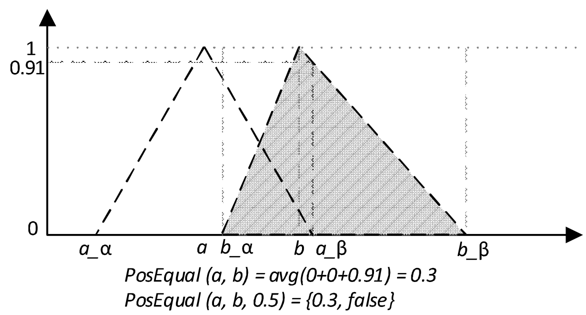

Fig. 15. Relation *Equals*: the value of time point "b" defined with a possible deviation

The degree of possible equality of "*a*" and "*b*" calculated using the average of *pos_extent* values:

PosEqual =AVG (pos_extent (a), pos_extent (a_α), pos_extent (a_β))

2.3) The value of time point "b" is not defined in the range

Possibilistic time point "*a*" with periods of a possible deviation of the occurrence *a_α* and *a_β*. Possibilistic time point "b" with possibilistic range of occurrence *b_α*, b_β, and possible degree *b_PExtent_IN* (Fig. 16).

For each $a\_x \in A$, where $A = \{a, a\_\alpha, a\_\beta\}$, performed the following calculation of *pos_extent* degree:

$$pos\_extent = \begin{cases} 0, & a\_x > b\_\beta \vee a\_x < b\_\alpha \\ 1 * b\_PExtent\_IN, & b\_\alpha \le a\_x \le b\_\beta \end{cases}$$

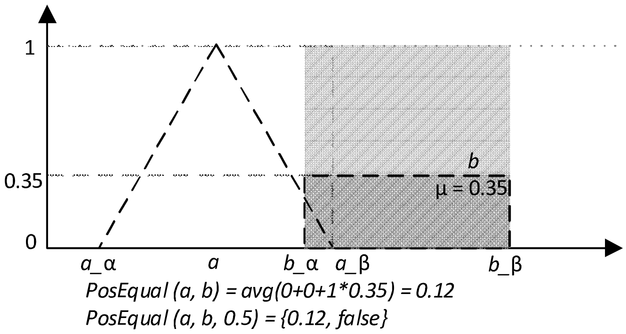

Fig. 16. Relation *Equals*: the value of time point "*b*" is not defined in the range

The degree of possible equality of "*a*" and "*b*" calculated using the average of *pos_extent* values:

PosEqual =AVG (pos_extent (a), pos_extent (a_α), pos_extent (a_β))

2.4) The value of time point "b" is not defined

Possibilistic time point "*b*" with unknown time of occurrence.

The relation returns the original value of *PExtent_IN*, which is equal to 0.

3) The value of time point "a" is not defined in the range

3.1) Crisp value of time point "b"

Possibilistic time point "*a*" with possibilistic range of occurrence *a_α*, a_β, and possible degree *a_PExtent_IN* (Fig. 17).

$$PosEqual = \begin{cases} 0, & b < a\_\alpha \vee b > a\_\beta \\ 1 * b\_PExtent\_IN, & a\_\alpha \le b \le a\_\beta \end{cases}$$

3.1) The value of time point "b" defined with a possible deviation

Possibilistic time point "*a*" with possibilistic range of occurrence *a_α*, a_β, and possible degree *a_PExtent_IN*. Possibilistic time point "*b*" with periods of a possible deviation of the occurrence *b_α* and *b_β* (Fig. 18).

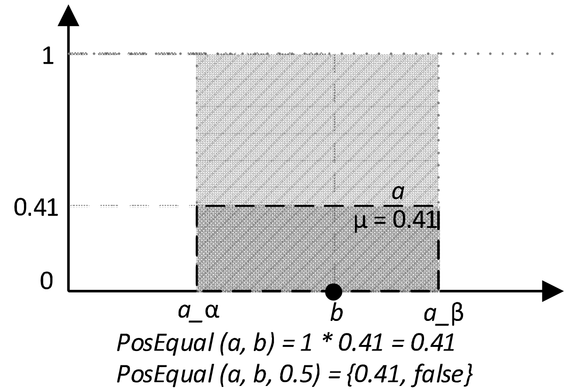

Fig. 17. Relation *Equals*: crisp value of time point "*b*"

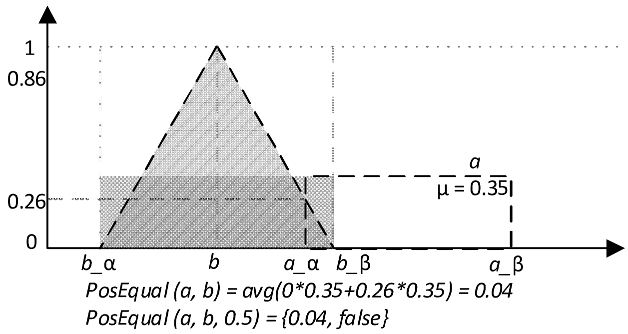

Fig. 18. Relation *Equals*: the value of time point "b" defined with a possible deviation

For each $a\_x \in A$, where $A = \{a\_\alpha, a\_\beta\}$, performed the following calculation of *pos_extent* degree:

$$pos\_extent = \begin{cases} 0, & a\_x \le b\_\alpha \vee a\_x > b\_\beta \\ \frac{a\_x - b\_\alpha}{b - b\_\alpha} * a\_PExtent\_IN, & b\_\alpha < a\_x \le b \\ \frac{b\_\beta - a\_x}{b\_\beta - b} * a\_PExtent\_IN, & b < a\_x \le b\_\beta \end{cases}$$

The degree of possible equality of "*a*" and "*b*" calculated using the average of *pos_extent* values:

PosEqual =AVG (pos_extent (a_α), pos_extent (a_β))

3.2) The value of time point "b" is not defined in the range

Possibilistic time point "*a*" with possibilistic range of occurrence *a_α*, a_β, and possible degree *a_PExtent_IN.* Possibilistic time point "*b*" with possibilistic range of occurrence *b_α*, b_β, and possible degree *b_PExtent_IN* (Fig. 19).

For each $a\_x \in A$, where $A = \{a\_\alpha, a\_\beta\}$, performed the following calculation of *pos_extent* degree:

$$pos\_extent = \begin{cases} 0, & a\_x > b\_\beta \vee a\_x < b\_\alpha \\ a\_PExt\_IN * b\_PExt\_IN), & b\_\alpha \le a\_x \le b\_\beta \end{cases}$$

The degree of possible equality of "*a*" and "*b*" calculated using the average of *pos_extent* values:

PosEqual =AVG (pos_extent (a_α), pos_extent (a_β))

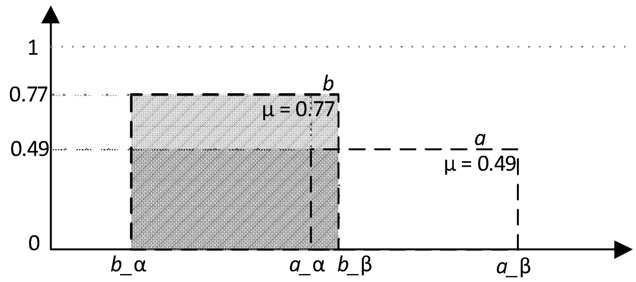

Fig. 19. Relation *Equals*: the value of time point "*b*" is not defined in the range

3.3) The value of time point "b" is not defined

Possibilistic time point "*b*" with unknown time of occurrence.

The relation returns the original value of *PExtent_IN*, which is equal to 0.

Let's apply possibilistic relation *Equals* to input possibilistic intervals:

PosEqual (IPeriod, JPeriod[, PExtent_IN])

where IPeriod and *JPeriod* – mandatory compared interval values, *PExtent_IN∈ [0,1]* (optional) – degree, required to fulfill the condition. The relation returns (*PExtent_OUT[, boolean]),* where *PExtent_OUT ∈ [0,1]* – the resulting possibilistic degree of fulfillment, *boolean*: *true*, if *PExtent_OUT* ≥ *PExtent_IN; false*, if *PExtent_OUT<PExtent_IN.*

In accordance with classical crisp relation *Equals* (Table I), possibillistic condition for the relation will look as follows:

*PosEqual (IPeriod.startdate, JPeriod.startdate) * PosEqual (IPeriod.enddate, JPeriod.enddate)*

C. The library of algorithms and model architecture

The algebra of relations between possibilistic time points and/or intervals has been implemented through a library of algorithms on the basis of pl/sql language. The developed library extends the capabilities of the SQL language for processing of fuzzy temporal information based on relational data models in the development of temporal data models theory. At present, more than 1300 lines of code are written. In the long term – the development of a single set of tools for database management systems to deal with temporal databases.

The algebra of relations between possibilistic time points and/or intervals was used for monitoring of service provider's billing platform dealing with loyalty points management system, including: forecasting of possibilistic completion dates of information aggregation in unified data collection system; retrieving and displaying information to subscribers about possibilistic date of premium packages activation.

Using temporal algebra of algorithms, we performed the analysis of billing system operations and their interactions with external systems. The analysis identified processes which contain uncertain and inaccurate information associated with time, including:

- Forecasting and storing possibilistic dates of payments performed by subscribers based on measurements of previous periods.

  Request example:

  "Save in database possible payment date: middle of May ([15.05.2016 10:00, 48:00:00, 48:00:00])"

- The analysis of time periods of subscriber payments, their comparison with possibilistic dates of forecasting reloads.

  Request examples:

  "Does the date of payment coincide every month (PosEqual (a, b))?"

  "Selection of subscribers with payments at the beginning of December ([01.12.2015 00:00, 03.12.2015 00:00, 0.6])"

- Prediction of possibilistic date of completion of a billing period cycle - collection of invoices and payments for the previous period.

  Request example:

  "Save in database possible date of receiving subscriber's invoice details: from 10 to 12 of June ([10.06.2016 00:00, 12.06.2016 00:00, 0.8])"

- Calculation and storing of possibilistic dates of premium packages activation given to subscribers for the usage of telecommunication services.

  Request example:

  "Save in database possible date of next bonus award: around 9th of April: ([00:45, 09.04.2015 12:32, 01:15])"

The algebra of algorithms between possibilistic time points and/or intervals has been applied to one of the above processes:

Telco operator provides to subscribers a monthly bonus packages type of "*25% discount for broadband services*". The ability to provide packages is calculated by a specialized loyalty platform in Operator's infrastructure based on the amounts of monthly payments performed by subscribers. Bonus packages are available once per month after reception of payment information from a billing system of the Operator.

Initial problem: In accordance with a public offer, the Operator shall provide bonus packages to subscribers on a monthly basis not later than the specified date. There is a restriction that does not allow the Operator to provide mentioned conditions: due to large volumes of data, providing of bonus packages happens with delays, since it is not known the exact date and time of the end of the billing period in the billing system, after which the information may be transferred to the loyalty platform. Delays in providing of packages lead to negative reviews and complaints from subscribers, thereby increasing churn rate.

Objective: to provide subscribers information on the estimated activation date of the bonus packages in accordance with the amount of their payments.

Solution: usage of the library of algorithms allowed to store possible date and time of payment information reception for all the customer base, allowing subscribers to receive information about possible bonus awarding date.

Implementation procedure: subscriber authenticates in Self-Service cabinet and opens "Bonus Program" page to check the reasons for the lack of bonus packages. Self-Service cabinet interacts with the loyalty platform via internal Operator's infrastructure for retrieving packages information of the subscriber. Due to lack of provided packages in the current month, loyalty platforms requests the subscriber's payment information from the billing system. In accordance with the amounts of data and performance measurements, billing returns possible end date of the billing period. Loyalty platform saves information in the database, returns the corresponding date to Self-Service cabinet and sends email and SMS notification to the subscriber informing him on expected date of a bonus package activation.

Result: subscriber checks information displayed on the Self-Service page and receives information by SMS/email. After that, subscriber waits for a specified period. Problem is solved.

The ongoing model architecture is shown on Fig. 20. *External System* is any system that directly deals with database which contains and handles possibilistic temporal information. In example above, it is Loyalty platform that saves information in its database, compares, retrieves it on demand of other external systems. An example of storing possibilistic date of data aggregation completion is shown on Table V.

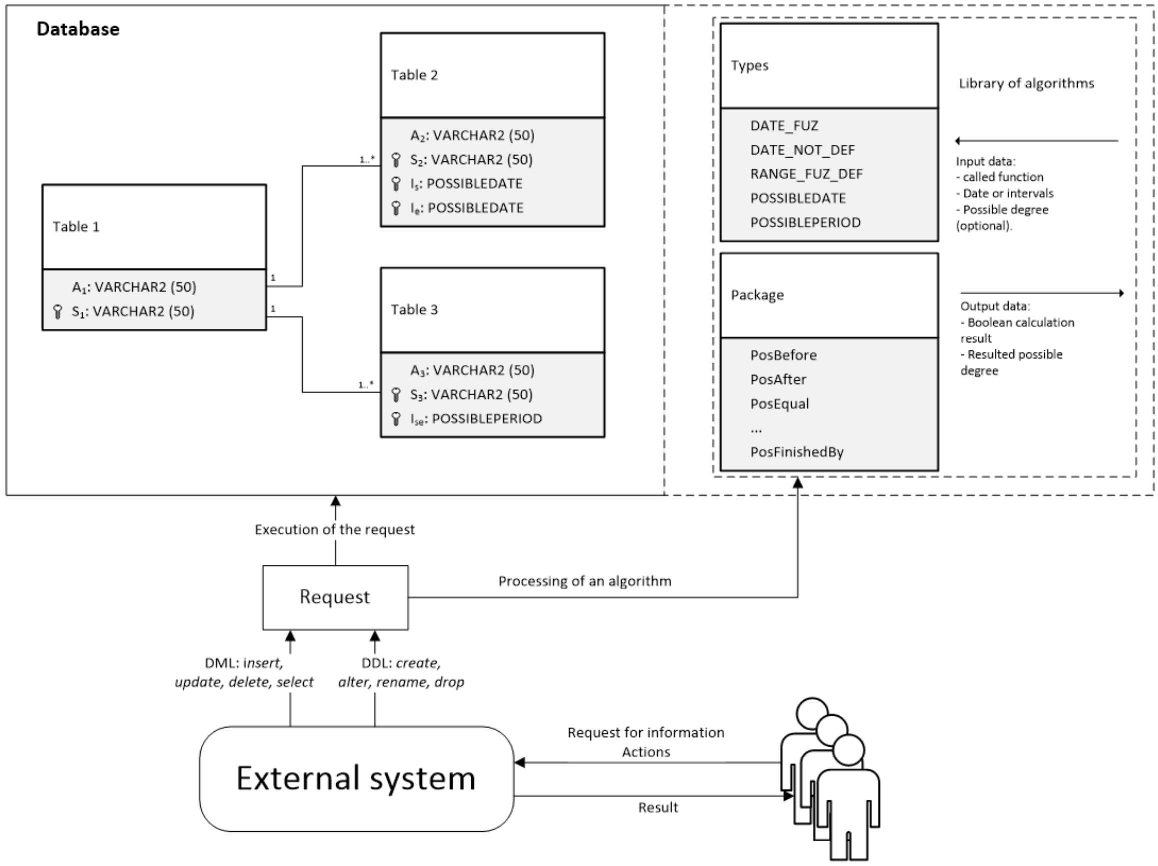

Fig. 20. General model architecture

TABLE V. TABLE IN DATABASE WITH POSSIBILISTIC TEMPORAL SUPPORT

P_ID	Period name	State	Pos_start_date	Pos_end_date
232	February 2016	Aggregation started	01.03.2016 00:00:00	09.03.2016 12:32:16
232	February 2016	Aggregation completed	09.03.2016 12:32:16	null
233	March 2016	Aggregation started	01.04.2015 00:00:00	[00:45, 09.04.2015 12:32, 01:15]

A model for storing and handling possibistic temporal information could be applied to any infrastructure and operations inside of it, which is subject to uncertainty. The model allows storing crisp and fuzzy temporal information, compare it and return possibilitic degree of conditions satisfaction.

First example could be [16] where besides of a qualitative information about the current state of the system (packet arrival rate of the incoming traffic or the degree of the output queue filling) there is an opportunity to store possibilistic date of the state change and deal with possibilistic time of receiving information due to delays between source systems.

Second example is [17] where mobile application gathers statistic about human health. Since at different moment of time disease can hold different states (low, high, stable) as described in [18], [19] and decease state change is not possible to predict in advance then there should be opportunity to store such kind of uncertain information with possibilistic temporal approach which allows to provide hypothetical results on the screen to end users.

Another example of applying a model is [20], where traffic prediction in wireless mesh networks was analyzed. In this work, one of the goals was to develop methods to predict system state in the future. Possibistic temporal approach allows to store fuzzy results of the prediction in the same fuzzy form, saving not only possibilistic states, but also possibilistic date and time of changing these states.

III. Conclusion

In this work we initiated the implementation of methods and algorithms for processing of temporal information which contains inaccuracy or uncertainty, as well as the approach to deal with possibilistic relations between time indicators. In further research, we plan to continue the construction of a new possibilistic-temporal model:

- Identify and design all the possible relations between possibilistic time points and intervals.
- Explore possibilistic non-temporal periods and applying of multiple possibilistic time axes in one relation.
- Implement the model for data definition and data manipulation languages (DML, DDL).
- Develop Java framework that allows to retrieve and compare possibilistic temporal information.

References

[1] R.T. Snodgrass, *Developing Time-Oriented Database Applications in SQL*, Morgan Kaufmann Publishers, Inc., San Francisco, 1999.

[2] J. Galindo, A. Urrutia, M. Piattini. *Fuzzy Databases: Modeling, Design and Implementation*, Idea Group Publishing, Hershey, USA, 2006.

[3] J. Galindo, *Handbook of Research on Fuzzy Information Processing in Databases*, Information Science Reference , Hershey, USA, 2008.

[4] C. Garrido, N. Marin, O. Pons, “Fuzzy intervals to represent fuzzy valid time in a temporal relational database”, *International Journal of Uncertainty, Fuzziness and Knowlege-Based Systems 17*(suppl. 1), 2009, pp. 173-192.

[5] G. Nagyp´al, B. Motik, “A fuzzy model for representing uncertain, subjective, and vague temporal knowledge in ontologies”, *On The Move to Meaningful Internet Systems 2003: CoopIS, DOA, and ODBASE*, 2003, pp. 906–923.

[6] N. Mahmood, A. Burney, K. Ahsan, “Generic Temporal and Fuzzy Ontological Framework”, *Journal of Universal Computer Science (JUCS)*, 2012, pp. 177-193

[7] J. Pons, C. Billiet, O. Pons, Guy De Tré, “Aspects of Dealing with Imperfect Data in Temporal Databases”, *Flexible Approaches in Data, Information and Knowledge Management,* 2014, pp. 189-220.

[8] J. Pons, N. Marín, O. Pons, C. Billiet, Guy De Tré, “A relational model for the possibilistic valid-time approach”, *International Journal of Computational Intelligence Systems*, 2012, pp. 1068-1088.

[9] J.M. Medina, J. Pons, C.D. Barranco, O. Pons, “A Fuzzy Temporal Object-Relational Database Model and Implementation”, *International Journal of Intelligent Systems,* vol. 29, 2014, pp. 836-863

[10] K.G. Kulkarni, J.E. Michels. “Temporal features in SQL: 2011”, *SIGMOD Record 41*(3), 2012, pp. 34-43.

[11] F. Zemke, “What’s new in SQL:2011”, *SIGMOD Record*, vol. 41, No. 1, 2012, pp. 67-73

[12] J.F. Allen, “Maintaining Knowledge about Temporal Intervals”. *Commun. ACM 26*(11), 1983, pp. 832-843

[13] H. J. Ohlbach, “Relations between fuzzy time intervals”, *International Symposium on Temporal Representation and Reasoning,* vol. 0, 2004, pp. 44-51

[14] D. Dubois, A. HadjAli, H. Prade, “Fuzziness and uncertainty in temporal reasoning”, *j-jucs*, vol. 9, 2003, pp. 1168-1194

[15] L.A. Zadeh, “The concept of a linguistic variable and its application to approximate reasoning”, I-III, Information Sciences 8, 1975, pp. 199–251

[16] V. Deart, A. Maslennikov, “Fuzzy Logic Queue Discipline Processing Over Bottleneck Link”, *Proceedings of the 11th Conference of Open Innovations Association FRUCT* , 2012, pp. 40-46

[17] A. Borodin, Y. Zavyalova, A. Zaharov, I. Yamushev, “Architectural Approach to the Multisource Health Monitoring Application Design”, *Proceedings of the 17th Conference of Open Innovations Association FRUCT*, 2015, pp. 16-21

[18] A Burney, N Mahmood, T Jilani, H Saleem, “Conceptual fuzzy temporal relational model (ftrm) for patient data”, *WSEAS Transactions on Information Science and Applications* 7 (5), 2010, pp. 725-734.

[19] A. Burney, N. Mahmood, Z. Abbas, “Advances in Fuzzy Rough Set Theory for Temporal Databases”, *Proceedings of the 11th WSEAS international conference on Artificial Intelligence, Knowledge Engineering and Data Bases*, 2012, pp. 237-242

[20] K. Krinkin, E. Kalishenko, "Traffic Prediction in Wireless Mesh Networks Using Process Mining Algorithms", Proceedings of the 11th Conference of Open Innovations Association FRUCT, 2012, pp. 88-94.