Digital Image Watermarking Using DWT Basis Matrices

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***Abstract*—This paper is devoted to image protection by means of computer steganography using wavelet techniques. Some research of the structure of the digital image is investigated, and we introduced its representation over basis images. Such basis images notation refers to orthogonal transform. We consider a set of wavelet basis images generated by orthogonal discrete wavelet transform and introduce a scheme for watermarking by embedding data to wavelet basis items.**

I. Introduction

The article is devoted to one of the most intensively developing areas of information protection — computer steganography. Modern steganography uses digital representation of data. Digital data can be different, like text, image, sound, and video. It is possible to embed one kind of data to another, leaving embedded data invisible, inaudible, i. e. not perceived by human sense organs. Yet embedded data is really hidden only for illegitimate users. The message presented in digital form may be placed into usual objects, which are transferred by communication channels. It can be used for transferring secret information. At the same time such invisible tags allow to solve various problems of information security, such as protection of copyright [1], development of secure e-services [2], and others.

Watermarking techniques have a number of applications including protection of digital media, forensic applications for labels, the color to gray and back problem and others. Most of these applications are intensively developed now because of the following reasons. Information is often represented in digital form and there are efficient tools for digital signal and image processing. Indeed, digital images are the most common type of media for which steganographic applications are currently available.

Following [3], digital steganography is a “science on invisible and foolproof hiding one bit sequences into another one, having analog nature”.

Among general problems of watermark embedding the problem of stability of stego image to transformations, which can lead to the embedded data distortion, is emphasized especially. Digital watermarks can be obtained in different ways and have various graphic representation. There are different well-known ways of watermarking, such as variants of Least Significant Bit (LSB), blocking and additive embedding, where bit presentation is used, embedding into Gray planes and others. An example of the steganographic system using Gray planes of grayscale digital image, which are based on the Gray codes, is described in [1]. There we applied modern halftoning algorithms (see details in [4], [5]) for obtaining digital binarized image, applicable as a digital watermark. The stego images are subjected to JPEG compression, and then accomplished the watermarks extracting with estimation of their degradation.

We use orthogonal transform of the digital grayscale image, formulated in terms of basis matrices. Orthogonal transforms are widely used in the image processing problems, particularly at lossy compression where discrete cosine transform and discrete wavelet transform (DWT) are used.

Most of the proposed techniques are devoted to embedding data directly into block of the coefficients, received after standard wavelet transform (see details in [6]–[8]). Note that there is also exist nonstandard wavelet type transform [9], [10] based on decomposition of the space of minimal splines [11].

The works [12]–[16] rely to embedding digital watermarks into digital grayscale images using DWT. The type of the wavelet can be different. These approaches can demonstrate the weakness of the proposed steganographic system. Also there can be used a combination of the DWT with other transform methods [14], [15]. For example, an approach with using DWT together with artificial neural network is presented in the paper [14]. In the work [15] some demands are made to digital watermark, then it transforms into pseudo-random binary sequence. One from the last work [16] devoted to embedding digital watermark based on block matching in DWT domain.

There are several approaches to embedding digital watermark using standard DWT with decomposition into coefficients. The standard DWT technique which is cited in literature sources refers to the frequency domain and digital data are usually embedded in the wavelet coefficients. For embedding digital watermarks into digital image at Print-Scan transform several techniques are known, for example Color to Gray and Back Reversible Transformation is presented in [17].

The well-known approaches of building the steganographic system based on DWT differ from each other in having secrete key, embedding techniques, preliminary digital watermark and cover work processing. However, the ways of the digital watermark embedding with a help of DWT with decomposition into basis matrices in literature resources are not presented. In the next sections we will describe statements of the orthogonal transform, basis DWT images and propose two embedding schemes of watermarks using suggested decomposition of the digital image onto basis images and coefficients.

Embedding of the digital watermarks is possible to divide into two groups: embedding into primary image domain or spatial domain and embedding into transform domain or frequency domain. Embedding into spatial domain operates separate elements of the image — pixels or block of pixels. DWT belongs to frequency domain transformation. These methods are often more stable to the distortions, but have several features. In particular, embedding into frequency domain is much more exigent to the resources. Besides that, these methods are much more complicated in the realization and need to take into consideration many peculiarities.

The proposed scheme uses the wavelet basic images for watermarking and can not be considered as a widely spread frequency or spatial domain technique. Indeed our algorithm replaces a part of the wavelet coefficients with message. A close algorithm was discussed for the protection of PDF documents (see [18]). It was found that such algorithms have detection errors due to calculation of DWT. The reason is the digital storage. Image using a graphical format needs integer encoding. It results in lost of accuracy because of data averaging after evaluation.

II. Basis images

Orthogonal transform of a grayscale image can be represented by a set of matrices that called *basis* images.

Consider a grayscale image presented by a $M \times N$ matrix F and two orthogonal matrices U and V with the size $M \times M$ and $N \times N$ respectively, where

$$UU^T = U^T U = 1,$$
$$VV^T = V^T V = 1.$$

Orthogonal transform of F has the form

$$\begin{array}{l} F = UGV^T, \\ G = U^T FV, \end{array} \tag{1}$$

where the matrix G has the size $M \times N$. Sometimes the matrix G is called the *representation* of F.

A matrix with its elements are assumed to be real, and we denote it by $A = \{A[i, j]\},\ i = 1, \dots, M,\ j = 1, \dots, N$. Let us introduce column vectors of the two orthogonal matrices accordingly

$$u_k = (U[1, k], U[2, k], \dots, U[M, k]),$$
$$v_p = (V[1, p], V[2, p], \dots, U[N, p]),$$

and define basis images as a tensor product

$$E_{(kp)} = u_k \otimes v_p, \tag{2}$$

where $k = 1, \dots, M,\ p = 1, \dots, N$. Here we enumerate matrices by the lower subscript (kp) in brackets to avoid confusing with matrix elements. From the definition it follows that the matrix elements of $E_{(kp)}$ are product of the U and V elements

$$E_{(kp)}[x, y] = U[x, k]V[y, p]. \tag{3}$$

In general, it will be MN basis images; each of them is a real matrix with size $M \times N$. They are orthonormal, i. e.

$$(E_{(kp)}, E_{(ab)}) = \delta_{ka}\delta_{pb}, \tag{4}$$

where scalar product of two matrices is a sum of the products of their elements of the same name, δ_{ij} is the Kronecker symbol. Using (2) we obtain a decomposition of the grayscale image F into the introduced basis images

$$F = \sum_{kp} G[k, p]E_{(kp)}, \tag{5}$$
$$G[k, p] = (F, E_{(kp)}),$$

where we can consider $G = \{G[k, p]\}$ as "coordinates" of F. We consider F as an image in spatial domain, that is image as we see it, then G will be image in frequency domain. The representation (5) is a form of the orthogonal transformation. Indeed, it follows from (1) written for matrix elements

$$\begin{aligned} F[x, y] &= \sum_{kp} U[x, k]G[k, p]V[y, p] \\ &= \sum_{kp} G[k, p]E_{(kp)}[x, y] = G[1, 1] \cdot E_{(11)}[x, y] \\ &+ G[1, 2] \cdot E_{(12)}[x, y] + G[2, 1] \cdot E_{(21)}[x, y] + \dots. \end{aligned}$$

One can see that element $F[x, y]$ is represented as a linear combination of the matrix elements of the matrices $E_{(11)}$,$E_{(12)}$, $E_{(21)}, \dots$.

Together with second equation in (1) we can introduce a basis images set created by tensor product of the orthogonal matrix U rows

$$J_{(xy)} = U[x, :] \otimes V[y, :]. \tag{6}$$

Using this set we get representation of G in the form

$$G = \sum_{xy} F[x, y]J_{(xy)},$$

where

$$F[x, y] = (G, J_{(xy)}).$$

III. Basis matrices and their properties

In this section we will discuss properties of the basis matrices. We focus on a particular case $U = V$ that is more interesting for practice. Being tensor product of the orthonormal vectors both sets $\{E_{(kp)}\}$ and $\{J_{(xy)}\}$ have similar properties.

The principal properties are the following.

1) Orthogonal transform of the grayscale image can be represented as a decomposition over the basis images.
2) Two basis images sets can be obtained by tensor product of the columns or row vectors of the orthogonal matrix U.
    - The basis images are representation of the operators designing type. In the quantum mechanics these operators describe, for example, the transition of particle between the discrete states.
3) The grayscale image can be decomposed into a set of the coefficients and the basis images set (Fig. 1).

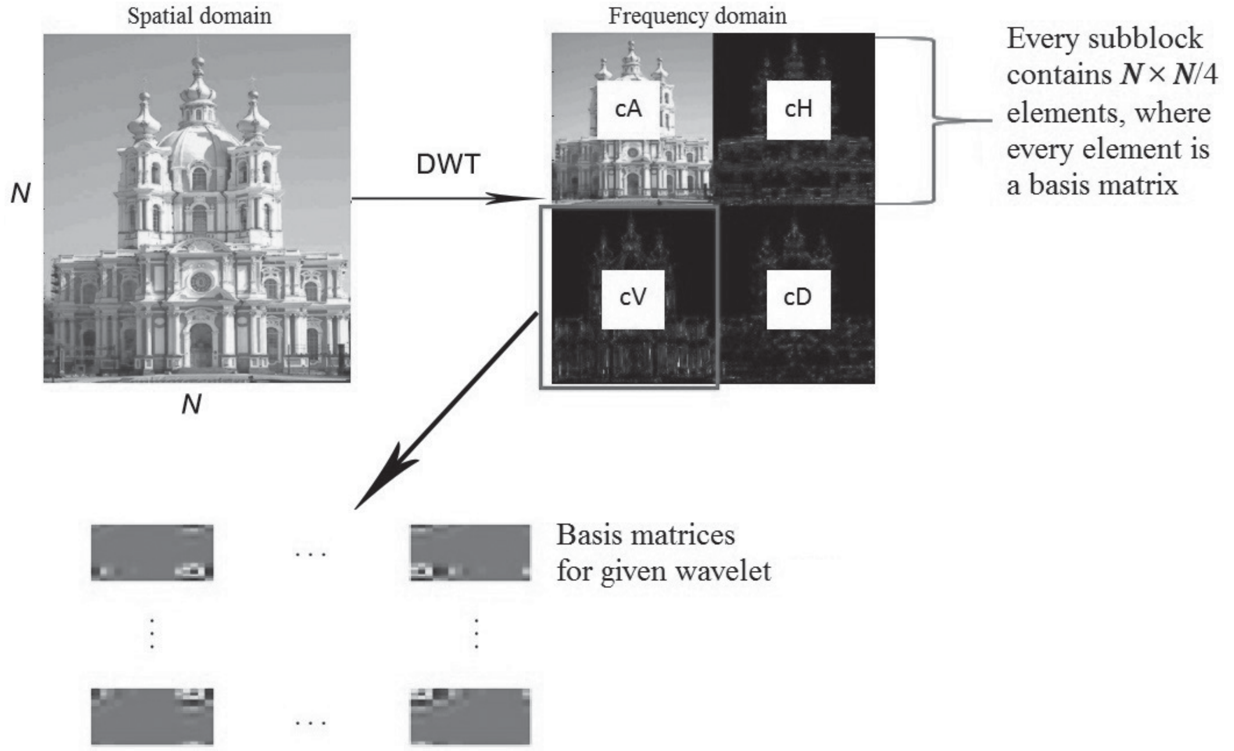

Fig. 1. Transform of the digital image into set of coefficients and set of basis images

4) Two sets of basis images allow to represent the grayscale image in spatial and frequency domain.
5) For the particular case of the orthogonal wavelet transform on the practice, developed calculations are built without using orthogonal matrix U in explicit form.
6) The general properties of the basis images allow to build matrices E and J, using developed calculation technique.
7) According with a block structure of the wavelet coefficients cA, cH, cV, cD, the set of the basis functions type E also has a block structure.
8) The block structure of E corresponds to representation of the image in spatial domain as a sum of the approximation and detail coefficients: $A+H+V+D$.
9) Using representation of the image as a set of the coefficients and basis images, presented at Fig. 1, it is possible to embed the data not only into coefficients, but also into basis matrices.

The properties of the basis images follow below.

A. Matrix product

For two basis images we have

$$E_{(ab)} \cdot E_{(cd)} = E_{(ad)}\delta_{bc}.$$

B. Orthogonal transform of the basis images

What will happen if the basis images are transformed orthogonally?

Let $E_{(ab)} = F$, then from (5) we found

$$G[k,p] = (E_{(ab)}, E_{(kp)}) = \delta_{ka}\delta_{pb}.$$

The obtained matrix G has all its elements equal to zero, except of the one of them, it is equal to 1 and placed in the position $k = a$, $p = b$. It means that orthogonal transform of the basis image results in a binary matrix of unit brightness. For such matrix we introduce notation $\mathbb{I}_{(ab)} = \{A[m,n]\}$, were all matrix elements are equal to zero except one: $A[m,n] = \delta_{ma}\delta_{nb}$.

It is possible to assert that basis images are the result of the orthogonal transform of the binary matrices with a unit brightness.

$$\begin{aligned} E_{(ab)} &= U\mathbb{I}_{(ab)}U^T, \\ \mathbb{I}_{(ab)} &= U^T E_{(ab)} U. \end{aligned} \quad (7)$$

C. Decomposition of the binary matrix having unit brightness

Decomposition $\mathbb{I}_{(ab)}$ of basis images is obtained in a form

$$\mathbb{I}_{(ab)} = \sum_{k,p} E_{(kp)}[a,b]E_{(kp)}$$

$$= E_{(11)}[a,b]E_{(11)} + E_{(12)}[a,b]E_{(12)} + \ldots,$$

where $E_{(kp)}[a,b]$ is brightness of pixel in position a, b of the basis image $E_{(kp)}$.

D. Product with matrices of unit brightness

There are two properties connected with matrix and scalar product.

$$\mathbb{I}_{(ab)} \cdot A \cdot \mathbb{I}_{(bc)} = A[b, c] \cdot \mathbb{I}_{(ab)},$$

$$(\mathbb{I}_{(ab)}, A) = A[a, b].$$

An example. Translation through communication channel. The basis images are called the *coding book*. It allows to decrease quantity of the information coming through channel. Let image F needs to be transferred from point A to point B. Instead of sending the image F directly into channel, an image $E_{(kp)}$, which is the most similar to F, is found in the coding book.

Similarity criteria can be for example the value of the coefficients G_{kp} in decomposition (5), Euclidean distance or something else. As a result the index (kp) is transferred into the channel. This approach leads to economy. Indeed, to transfer halftone 8-bit digital image with a size $M \times N$ it is necessary to send into channel $H(F) = 8MN$ bits. And to transfer one index from coding book, having MN basis images, it needs $\log_2 MN$ bits, that is less then $H(F)$. The back side of this economy is a loss of quality, which can be compensated, if several basis images are transferred.

IV. Basis DWT images

Here we assume that $U = V$, where U is a $N \times N$ orthogonal DWT matrix. Matrix U has a block structure that results in the block structure of DWT coefficients and basis images. The orthogonal matrix U allows to transform the grayscale image from spatial domain into frequency domain and back. In the case of a one-level DWT, matrix U consists of two $N/2 \times N$ matrices L and H, known as Low and High frequency bands. As results we get the DWT coefficients as

$$G = DWT(F) = \begin{bmatrix} cA & cH \\ cV & cD \end{bmatrix},$$

and

$$F = IDWT(G),$$

where $IDWT$ is the inverse transform. The $N/2 \times N/2$ matrices are called *approximation* cA, *horizontal* cH, *vertical* cV and *diagonal* cD *details*, known also as LL, LH, HL and HH frequency bands.

Basis images can be calculated as $E_{(kp)} = U\mathbb{I}_{kp}U$. Note that indexes (k, p) may belong to one of the blocks cA, cH, cV and cD. Let $(k, p) \in cH$ then we have a set of basis images

$$E_{(kpH)} = IDWT \begin{bmatrix} 0 & \mathbb{I}_{kp} \\ 0 & 0 \end{bmatrix},$$

where $k, p = 1, \ldots, N/2$. Total number of items $E_{(kpH)}$ is $N^2/4$, each item is a $N \times N$ matrix.

Using the set of $E_{(kpH)}$ and the cH block coefficients an approximation of F can be achieved

$$H = \sum_{k,p} cH[k, p]E_{(kpH)}.$$

The block structure of E corresponds to representation of the image in spatial domain as a sum of the approximation and detail coefficients. So, we can represent any image as a sum

$$F = A + H + V + D, \tag{8}$$

where A is an approximation of F, and other matrices refer to details.

It follows that the considered basis is represented by four blocks of orthonormal items as

$$\Big(\{E_{(kpA)}\}, \{E_{(kpH)}\}, \{E_{(kpV)}\}, \{E_{(kpD)}\}\Big).$$

So in accordance with (8) the representation of grayscale image F takes the form

$$F = \sum_{kp} cA[k, p]E_{(kpA)} + cH[k, p]E_{(kpH)} + cV[k, p]E_{(kpV)} + cD[k, p]E_{(kpD)}.$$

We can create the second set given by (6). The wavelet coefficient block structure produces such structure of each items from the set

$$J_{(xy)} = \begin{bmatrix} J_{(xyA)} & J_{(xyH)} \\ J_{(xyV)} & J_{(xyD)} \end{bmatrix}.$$

Using representation of the image as a set of the coefficients and basis images set, that is shown at Fig. 1, it is possible to embed the data not only to coefficients, but to basis matrices.

V. A steganographic scheme

The introduced representation allows to embed a message into cover image by modification of an item from the basis, and orthogonal DWT can be used.

A. Obtaining of the basis DWT images

For our scheme the firstly it needs to generate a set of basis wavelet matrices. It can be achieved using (7). An example of a basis image is shown at Fig. 2.

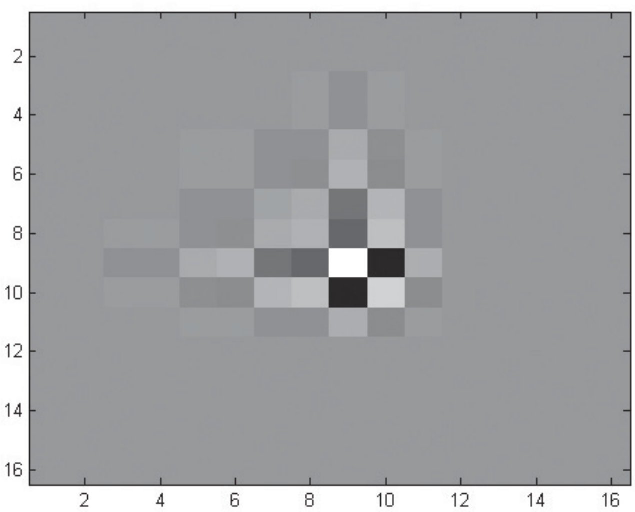

Fig. 2. Basis matrix $E_{(3,3,4)}$ refers to diagonal block cD, 'db6'

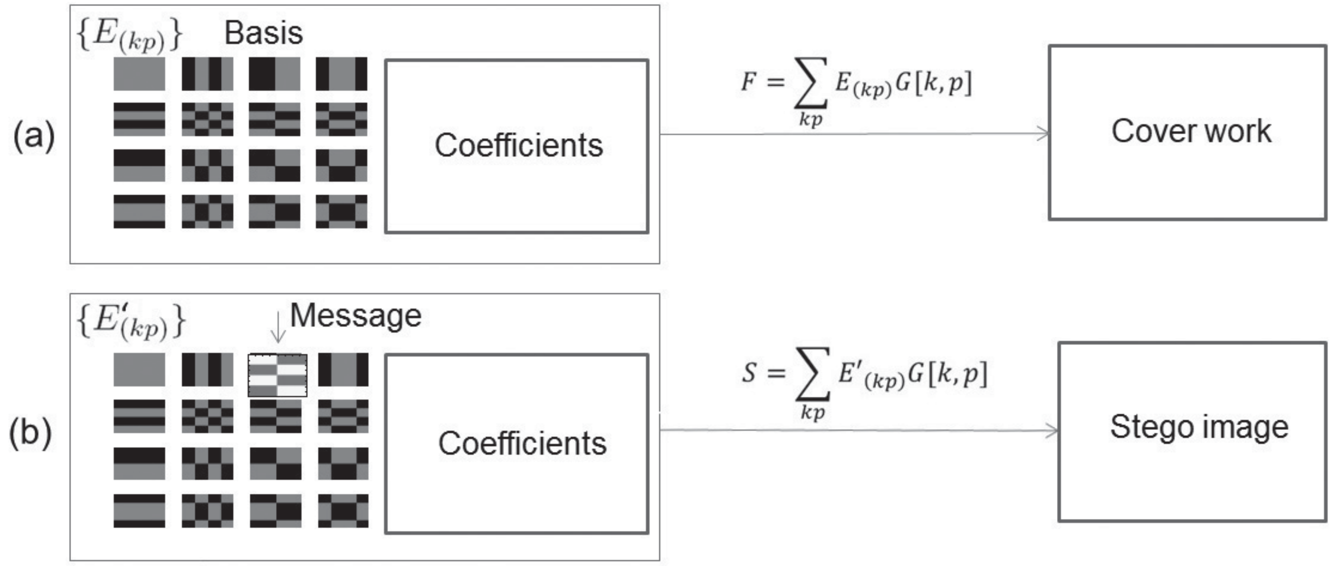

Fig. 3. The embedding scheme. (a) Representation of the cover image using basis matrices and coefficients; (b) replacing of the basis item

The image is obtained by one-level DWT of the 16×16 grayscale image using the Daubechies wavelet 'db6' (see [19]). This is the one of the image from block referred to cD coefficients. This block has $N^2/4 = 64$ images of size 16×16.

B. Embedding algorithm

Our scheme is based on modification of basis images that presents a grayscale cover work. Algorithm has the following steps.

1) Choose an orthogonal matrix U and represent a cover grayscale image F by its coefficients $G[k,p]$ and a set of the basis images $A = \{E_{(kp)}\}$ generated from U.
2) Replace an item from A with a new item that has embedded data. The obtained set A' is not a basis.
3) Get the stego image S using coefficients $G[k,p]$ and images from A'.

Relation between initial basis and modification. Let the matrix A consists of the column-vectors

$$A = [a_1, \dots, a_t, \dots, a_n].$$

Let matrix $A' = [a'_1, \dots, a'_t, \dots, a'_n]$ consists of the elements of the matrix A except one replaced column-vectors a_t^M, i. e.

$$A' = [a_1, \dots, a_t^M, \dots, a_n].$$

If the vector a_t^M can be represented as

$$a_t^M = \sum_{k=1}^{n} x_k a_k,$$

then the following relation is valid

$$A' = AQ, \tag{9}$$

where the matrix Q consists of scalar products

$$Q[k,p] = (a_k, a'_p),$$

and has the following structure

$$Q[k,p] = \begin{cases} \delta_{kp}, & k, p \neq t, \\ x_k, & p = t. \end{cases}$$

It is equivalent to equality

$$Q[k,p] = \delta_{kp}(1 - \delta_{pt}) + x_k \delta_{pt}.$$

It follows that

$$a'_p = a_p(1 - \delta_{pt}) + \sum_{k=1}^{n} x_k a_k \delta_{pt}.$$

Inverse matrix Q^{-1} has analogous structure. It is square matrix with a column t, defined by its coordinates $x_1, \dots, x_n$,

$$Q^{-1}[:,t] = -\frac{1}{x_t}(x_1, \dots, x_{t-1}, -1, x_{t+1}, \dots, x_n).$$

The embedding scheme is presented at Fig. 3.

Computational complexity is one of important features of an algorithm for its application. Our scheme includes the orthogonal transform and calculates basic images that are tensor products of vectors. All operations are arithmetics and perform over the n-bit data. Then they have polynomial complexity and our algorithm belongs to P-class.

Detection steps depend on transformation of a chosen basis image $E_{(cd)}$ into a new item $E_{(cd)M}$ that includes message M. Since $E_{(cd)}$ is a graysale image, we can use a standard image based watermarking algorithm EM

$$E_{(cd)} \to E_{(cd)M} = EM(E_{(cd)}, M).$$

In general case for detection it needs the cover image C, matrix U and position (c,d), that can play role of a secrete key.

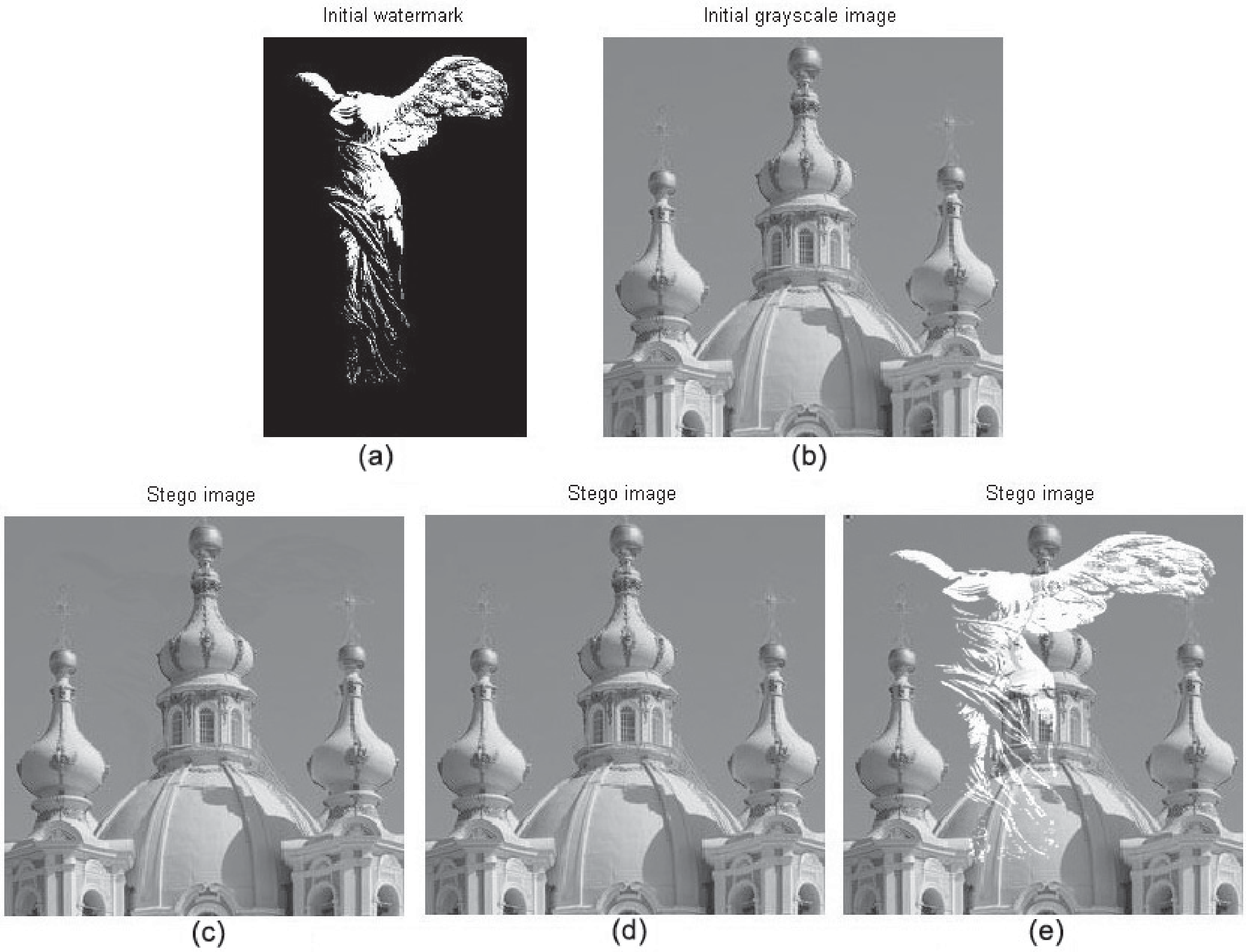

Fig. 4. Watermark embedding. (a) Binary watermark; (b) cover work; (c) stego image for $E(3,3,4)$ without changing brightness of the watermark; (d) stego image with invisible watermark (e) stego image with visible watermark

C. Experiment

In the case of DWT the basis images set has four blocks that refer to DWT coefficients. It can be taken into account by introducing the third index $k = 1, 2, 3, 4$ to label the basis items associated with cA, cH, cV and cD blocks of the DWT coefficients. So $E_{(cd4)}$ is a $N \times N$ matrix of the cD basis set block, that has $N^2/4$ items and $c, d = 1, \ldots, N/2$.

Our embedding algorithm replaces a chosen basis image $E_{(cdk)}$ with a message that is a binary image M. The algorithm has the following parameters: $\{w, E_{(cdk)}, a\}$, where w is an orthogonal wavelet, a is a scaling factor to change the brightness M. Visible and invisible watermark can be achieved by varying of factor a.

Fig. 4 illustrates the work of our embedding algorithm. The binary watermark presented at Fig. 4(a) is embedded into cover work (Fig. 4(b)) using wavelet 'db6'. The firstly, watermark is resized and its brightness is scaled $M \to aM$, where a can be found from experiment only. Visibility of the watermark depending on its brightness and parameter k. So, for $k = 4$ (response for diagonal details) we can set any the value $a < 1$ and receive invisible watermark. This example is shown at Fig. 4(d) where stego image and cover image are undistinguished visually.

It results in the fact that the chosen basis image $E(3,3,4)$ has a small weight and introduces a small changing into cover work. In contrast the basis item $c = 3$, $d = 4$, $k = 1$ can introduce large changing. This case presented at Fig. 4(e), where $a = 1/500$, that means we reduced the watermark brightness by 500 times. The chosen parameters lead to visible watermark.

VI. Conclusions

In the paper the investigation of the structure of the digital image is done. We propose the approach of constracting the basis matrices and consider the possibilities of their using for the task of the digital data embedding. Our technique means representation of the digital image by means of set of the usual wavelet coefficients and some basis images, defined by wavelet transform. Proposed steganographic system of digital watermark embedding into digital grayscale image allows to embed digital data by means of replacing the constructed basis wavelet image for chosen block of the coefficients. Using this approach we can manipulate of the choosing of the block, thus we can influence on visibility of the watermark.

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