Automated Robotic System with Five Degrees of Freedom

Sergei Ivanov, Tatiana Zudilova, Irina Osetrova, Igor Anantchenko, Andrei Mikalauskas
ITMO National Research University (ITMO University)
Saint Petersburg, Russian Federation
Serg_ie@mail.ru, Zudilova@ifmo.spb.ru, Irina@ifmo.spb.ru, Anantchenko@yandex.ru

Abstract—**In paper develops and studies a mathematical model of an industrial robot with five degrees of freedom. The kinematic scheme of the robot is determined and the development of a mathematical model is performed using the matrix method and the Lagrange matrix dynamic equations. The study of the model is performed using the method of polynomial transformations. As a result of the analysis of the mathematical model, generalized coordinates, velocities and accelerations for the links of the robot are obtained. The spatial trajectory of the robot's gripper relative to the fixed base of the stand in the initial coordinate system is presented. The developed mathematical model of the robot allows you to determine the main dynamic characteristics of the robot, as well as the necessary generalized forces created by the drive links to move the robot's gripper to the selected point. This model allows the development of automated robotic systems and can be used in robot control systems with five degrees of freedom.**

I. Introduction

Currently, the urgent task is the development and study of models of robotic systems for industrial automation. A large number of scientific literatures devoted to the problems of simulation of robots.

In paper [1] considers the problem of kinematic and dynamic trajectory tracking control for a mobile robot. The article performs modeling and experiment to verify and confirm the proposed control strategy.

In paper [2], a modeling method is presented for moving and manipulating a robot. The article presents a new method for generating contacts, which considers geometry as the presence of a thin virtual boundary layer around the underlying grids. The simulator is tested on various examples of robot movements, and the results are consistent with theoretical predictions and experimental data.

In paper [3], the task of controlling a robot tracker with machine vision capabilities is considered. The article proposes an optimal control scheme based on a genetic algorithm. A genetic algorithm is used to obtain optimal values of control constants based on a suitable cost function. As a result of the experiments, the effectiveness of a visual robot was determined, including an optimal controller for tracking target trajectories and confirming the stability of the control system.

The book [4] deals with the problem of simulating robots, modeling the dynamics of robots, and controlling robots. Simulators are based on CAD and graphical visualization tools. The simulation allows you to design and test robots in various environments for a small time and cost to create real systems.

In paper [5] discusses a platform for autonomous programming based on CAD (OLP), which allows you to generate a robot path from a CAD model and visualize the simulation graphically. Based on the platform, a simulation of an industrial manipulator with 6 degrees of freedom is performed.

In paper [6] performs the development and analysis of the structure, which allows representing the physical effects of the manipulation action of robots. A qualitative method of reasoning is considered, which argues on actions and their consequences based on projection-based modeling. The method allows the robot to determine what can happen if it performs the task in a certain way. Using this method, robots can perform manipulation tasks more efficiently, reliably, and flexibly.

In paper [7] proposes adaptive fractional order PID sliding mode controller for Caterpillar robot manipulator. Sliding mode controller is one of the control methods which provide high robustness and low tracking error. The new combined control law is proposed for chattering reduction by means of fractional order PID controller and trajectory tracking through using sliding mode controller.

In paper [8] a robust nonlinear back stepping technique with Lagrange's extrapolation and PI compensator is proposed for high accuracy trajectory tracking of robot manipulators with uncertain dynamics and unexpected disturbances. The proposed control technique shows better performances via experimental results on a 7-DOF robot arm in comparison with the classical back stepping and sliding mode control.

In paper [9] develops a new kinematic scheme for a robotic lift. The dynamic equations of the lift motion are derived and analyzed by the analytical method.

In our work, we are developing and exploring a dynamic model of an industrial robot with five degrees of freedom to obtain the dynamic characteristics of the robot.

The resulting basic dynamic characteristics of the robot, such as coordinates, speeds and accelerations, allow the development of an automated robot control system.

ISSN 2305-7254

II. Mathematical model of an industrial robot with five degrees of freedom

Consider an industrial robot with five degrees of freedom, which includes the main links: the base, stand, arm, actuator - gripper and drives for extension and turns.

For the robot circuit under consideration, the links are represented by rods connected by cylindrical hinges and sliding joints. We assume that the friction in the joints is small and is not taken into account when deriving the robot model.

The kinematic scheme of the robot is shown in Fig. 1 and consists of four rotational kinematic pairs and one translational pair. The robot model is applicable to MP industrial manipulators, the kinematic diagram of which is shown in Fig. 1.

When developing the dynamic equations of a robot, we apply the matrix method and Lagrange matrix dynamic equations. In the matrix method, extended transition matrices from one coordinate system of the robot links to another coordinate system are used. Following [10], a special choice of coordinate systems for robot links is applicable. As a result, the transition from one coordinate system to one is determined not by six, but by four parameters. To move from one coordinate system to the next coordinate system, need a turn around the axis, two shifts (translations along the axes) and another turn.

Define the coordinate system of the links of the robot in points O_1, O_2, O_3, O_4, O_5. The initial coordinate system is associated with the fixed base of the rack at the point O_0.

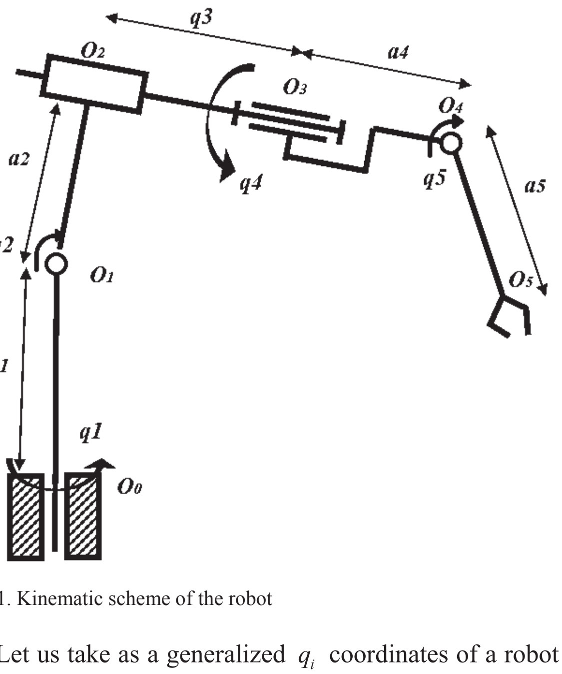

Fig. 1. Kinematic scheme of the robot

Let us take as a generalized q_i coordinates of a robot with five degrees of freedom: the angle of rotation around the rack, the angle of the rack, the length of the extension of the arm, the angle of rotation of the arm, the angle of rotation of the gripper. Here we measure angles in radians, lengths in centimeters.

To move from the O_0 coordinate system to the O_1 coordinate system, the following is required: rotation around the z axis by the angle q_1, shift along the z axis by the value of a_1, and rotation around the x axis by the angle –π / 2.

To move from the O_1 coordinate system to the O_2 coordinate system, the following is needed: rotation around the z axis by an angle of -π / 2, a shift along the z axis by an amount of a_2, and rotation around the x axis of an angle q_2.

To move from the O_2 coordinate system to the O_3 coordinate system, the following is required: a shift along the x axis by the q_3.

To move from the O_3 coordinate system to the O_4 coordinate system, the following is needed: a shift along the x axis by an a_4 value and a turn around the x axis by an angle of q_4.

To move from the O_4 coordinate system to the O_5 coordinate system, the following is required: rotation around the z axis by an angle of -π / 2, a shift along the z axis by an amount of a_5, and rotation around the x axis by an angle of q_5.

We introduce an extended radius – vector of points O_i, , in the *i*–th coordinate system:

$$R_i = \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}^T .$$

The transition matrix $A_{i-1,i}$ connects the radius – vector in the coordinate systems i – 1 and i: $R_{i-1} = A_{i-1,i} R_i$

Denote the functions: $S_i = Sin[q_i], C_i = Cos[q_i]$.

The transition matrices in the following coordinate system are defined:

$$A_{01} = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0 & C_2 & -S_2 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & S_2 & C_2 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{23} = \begin{bmatrix} 1 & 0 & 0 & q_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{34}=\begin{bmatrix}1 & 0 & 0 & a_4\\ 0 & C_4 & -S_4 & 0\\ 0 & S_4 & C_4 & 0\\ 0 & 0 & 0 & 1\end{bmatrix},$$

$$A_{45}=\begin{bmatrix}0 & C_5 & -S_5 & 0\\ -1 & 0 & 0 & 0\\ 0 & S_5 & C_5 & a_5\\ 0 & 0 & 0 & 1\end{bmatrix}.$$

The transition matrices from the fixed O_0 coordinate system to the O_i coordinate system are defined as the product of the transition matrices: $A_{0i}=A_{01}A_{12}\ldots A_{i-1\ i}$

$$A_{02}=\begin{bmatrix}0 & C_1C_2-S_1S_2 & -C_2S_1-C_1S_2 & -a_2S_1\\ 0 & C_2S_1+C_1S_2 & C_1C_2-S_1S_2 & a_2C_1\\ 1 & 0 & 0 & a_1\\ 0 & 0 & 0 & 1\end{bmatrix},$$

$$A_{03}=\begin{bmatrix}0 & C_1C_2-S_1S_2 & -C_2S_1-C_1S_2 & -a_2S_1\\ 0 & C_2S_1+C_1S_2 & C_1C_2-S_1S_2 & a_2C_1\\ 1 & 0 & 0 & a_1+q_3\\ 0 & 0 & 0 & 1\end{bmatrix},$$

$$A_{04}=\left[\begin{matrix}0 & C_4\left(C_1C_2-S_1S_2\right)+\left(-C_2S_1-C_1S_2\right)S_4\\ 0 & C_4\left(C_2S_1+C_1S_2\right)+\left(C_1C_2-S_1S_2\right)S_4\\ 1 & 0\\ 0 & 0\end{matrix}\right.,$$

$$\left.\begin{matrix}C_4\left(-C_2S_1-C_1S_2\right)-\left(C_1C_2-S_1S_2\right)S_4 & -a_2S_1\\ C_4\left(C_1C_2-S_1S_2\right)-\left(C_2S_1+C_1S_2\right)S_4 & a_2C_1\\ 0 & a_1+a_4+q_3\\ 0 & 1\end{matrix}\right],$$

$$A_{05}=\left[\begin{matrix}-C_4\left(C_1C_2-S_1S_2\right)-\left(-C_2S_1-C_1S_2\right)S_4\\ -C_4\left(C_2S_1+C_1S_2\right)-\left(C_1C_2-S_1S_2\right)S_4\\ 0\\ 0\end{matrix}\right.,$$

$$\begin{matrix}\left(C_4\left(-C_2S_1-C_1S_2\right)-\left(C_1C_2-S_1S_2\right)S_4\right)S_5\\ \left(C_4\left(C_1C_2-S_1S_2\right)-\left(C_2S_1+C_1S_2\right)S_4\right)S_5\\ C_5\\ 0\end{matrix},$$

$$\begin{matrix}C_5\left(C_4\left(-C_2S_1-C_1S_2\right)-\left(C_1C_2-S_1S_2\right)S_4\right)\\ C_5\left(C_4\left(C_1C_2-S_1S_2\right)-\left(C_2S_1+C_1S_2\right)S_4\right)\\ -S_5\\ 0\end{matrix},$$

$$\left.\begin{matrix}a_5\left(C_4\left(-C_2S_1-C_1S_2\right)-\left(C_1C_2-S_1S_2\right)S_4\right)-a_2S_1\\ a_2C_1+a_5\left(C_4\left(C_1C_2-S_1S_2\right)-\left(C_2S_1+C_1S_2\right)S_4\right)\\ a_1+a_4+q_3\\ 1\end{matrix}\right].$$

Denote the coordinates of the fingers of the gripper in the coordinate system O_5 for $\left(x_5,y_5,z_5\right)$, then in the fixed system O_0 the coordinates of the gripper have the form:

$$\begin{aligned}x_{05}=&-Sin[q_1]a_2+\\ &\begin{pmatrix}Cos[q_4]\left(-Cos[q_2]Sin[q_1]-Cos[q_1]Sin[q_2]\right)-\\ \left(Cos[q_1]Cos[q_2]-Sin[q_1]Sin[q_2]\right)Sin[q_4]\end{pmatrix}a_5+\\ &\begin{pmatrix}-Cos[q_4]\left(Cos[q_1]Cos[q_2]-Sin[q_1]Sin[q_2]\right)-\\ \left(-Cos[q_2]Sin[q_1]-Cos[q_1]Sin[q_2]\right)Sin[q_4]\end{pmatrix}x_5+\\ &\begin{pmatrix}Cos[q_4]\left(-Cos[q_2]Sin[q_1]-Cos[q_1]Sin[q_2]\right)-\\ \left(Cos[q_1]Cos[q_2]-Sin[q_1]Sin[q_2]\right)Sin[q_4]\end{pmatrix}Sin[q_5]y_5-\\ &\begin{pmatrix}Cos[q_4]\left(Cos[q_2]Sin[q_1]+Cos[q_1]Sin[q_2]\right)+\\ \left(Cos[q_1]Cos[q_2]-Sin[q_1]Sin[q_2]\right)Sin[q_4]\end{pmatrix}Cos[q_5]z_5,\end{aligned}$$

$$\begin{aligned}y_{05}=&Cos[q_1]a_2+\\ &\begin{pmatrix}Cos[q_4]\left(Cos[q_1]Cos[q_2]-Sin[q_1]Sin[q_2]\right)-\\ \left(Cos[q_2]Sin[q_1]+Cos[q_1]Sin[q_2]\right)Sin[q_4]\end{pmatrix}a_5+\\ &\begin{pmatrix}-Cos[q_4]\left(Cos[q_2]Sin[q_1]+Cos[q_1]Sin[q_2]\right)-\\ \left(Cos[q_1]Cos[q_2]-Sin[q_1]Sin[q_2]\right)Sin[q_4]\end{pmatrix}x_5+\\ &\begin{pmatrix}Cos[q_4]\left(Cos[q_1]Cos[q_2]-Sin[q_1]Sin[q_2]\right)-\\ \left(Cos[q_2]Sin[q_1]+Cos[q_1]Sin[q_2]\right)Sin[q_4]\end{pmatrix}Sin[q_5]y_5+\\ &\begin{pmatrix}Cos[q_4]\left(Cos[q_1]Cos[q_2]-Sin[q_1]Sin[q_2]\right)-\\ \left(Cos[q_2]Sin[q_1]+Cos[q_1]Sin[q_2]\right)Sin[q_4]\end{pmatrix}Cos[q_5]z_5,\end{aligned}$$

$$z_{05}=a_1+a_4+q_3+Cos[q_5]y_5-Sin[q_5]z_5.$$

We define the kinetic energy of all the links of the robot using the transition matrices using the formulas:

$$T_i=\frac{1}{2}tr\left(\dot{A}_{0i}H_i\dot{A}_{0i}^T\right), \tag{1}$$

Here H_i is the inertia matrix of the link.

$$H_i = \begin{bmatrix} J_{ixx} & J_{ixy} & J_{ixz} & m_i x_i \\ J_{iyx} & J_{iyy} & J_{iyz} & m_i y_i \\ J_{izx} & J_{izy} & J_{izz} & m_i z_i \\ m_i x_i & m_i y_i & m_i z_i & m_i \end{bmatrix},$$

where m_i is the mass of the link,

$J_{ixx}, J_{iyy}, J_{izz}$ – components of the inertia tensor of a link with respect to its own axes.

The coordinates of the centers of gravity of the link in the local coordinate system are indicated x_i, y_i, z_i.

Moments of inertia of the link relative to the axes are indicated J_{xi}, J_{yi}, J_{zi}.

Determine the kinetic energy of the links by the matrix formula (1) taking into account the equalities:

$$J_{iyy} + J_{izz} = J_{xi}, J_{ixx} + J_{izz} = J_{yi}, J_{ixx} + J_{iyy} = J_{zi}.$$

$$T_1 = 0.25 i_1^2 m_1 \left(q_1^{'}\right)^2,$$

$$T_2 = m_2 \begin{pmatrix} a_2 \left(0.5 a_2 + Cos[q_2] z_2\right)\left(q_1^{'}\right)^2 + \\ i_2^2 \left(0.25\left(q_1^{'}\right)^2 + 0.25\left(q_2^{'}\right)^2\right) \end{pmatrix},$$

$$T_3 = m_3 \begin{pmatrix} 0.5 a_2^2 \left(q_1^{'}\right)^2 + Cos[q_2] a_2 z_3 \left(q_1^{'}\right)^2 + \\ i_3^2 \left(0.25\left(q_1^{'}\right)^2 + 0.25\left(q_2^{'}\right)^2\right) + 0.5\left(q_3^{'}\right)^2 \end{pmatrix},$$

$$T_4 = 0.5 m_4 \begin{pmatrix} a_2^2 \left(q_1^{'}\right)^2 + 2.Cos[q_2 + q_4] a_2 z_4 \left(q_1^{'}\right)^2 + \left(q_3^{'}\right)^2 + \\ i_4^2 \left(0.5\left(q_1^{'}\right)^2 + 0.5\left(q_2^{'}\right)^2 + 0.5\left(q_4^{'}\right)^2\right) \end{pmatrix},$$

$$\begin{aligned} T_5 = {} & m_5 (0.5 a_2^2 \left(q_1^{'}\right)^2 + Cos[q_2 + q_4] a_2 \left(a_5 + Cos[q_5] z_5\right)\left(q_1^{'}\right)^2 + \\ & 0.5\left(q_3^{'}\right)^2 + a_5^2 (0.5\left(q_1^{'}\right)^2 + 0.5\left(q_2^{'}\right)^2 + 0.5\left(q_4^{'}\right)^2 + \\ & Cos[q_5] a_5 z_5 \left(\left(q_1^{'}\right)^2 + \left(q_2^{'}\right)^2 + \left(q_4^{'}\right)^2\right) + \\ & i_5^2 \left(0.25\left(q_1^{'}\right)^2 + 0.25\left(q_2^{'}\right)^2 + 0.25\left(q_4^{'}\right)^2 + 0.25\left(q_5^{'}\right)^2\right)) \end{aligned},$$

We write the total kinetic energy:

$$\begin{aligned} T = {} & 0.25 i_1^2 m_1 \left(q_1^{'}\right)^2 + Cos[q_2] a_2 \left(m_2 z_2 + m_3 z_3\right)\left(q_1^{'}\right)^2 + \\ & a_2^2 \left(0.5 m_2 + 0.5 m_3 + 0.5 m_4 + 0.5 m_5\right)\left(q_1^{'}\right)^2 + \\ & Cos[q_2 + q_4] a_2 \left(a_5 m_5 + m_4 z_4 + Cos[q_5] m_5 z_5\right)\left(q_1^{'}\right)^2 + \\ & \left(i_2^2 m_2 + i_3^2 m_3\right)\left(0.25\left(q_1^{'}\right)^2 + 0.25\left(q_2^{'}\right)^2\right) + \\ & \left(0.5 m_3 + 0.5 m_4 + 0.5 m_5\right)\left(q_3^{'}\right)^2 + \\ & i_4^2 m_4 \left(0.25\left(q_1^{'}\right)^2 + 0.25\left(q_2^{'}\right)^2 + 0.25\left(q_4^{'}\right)^2\right) + \\ & a_5^2 m_5 \left(0.5\left(q_1^{'}\right)^2 + 0.5\left(q_2^{'}\right)^2 + 0.5\left(q_4^{'}\right)^2\right) + \\ & Cos[q_5] a_5 m_5 z_5 \left(\left(q_1^{'}\right)^2 + \left(q_2^{'}\right)^2 + \left(q_4^{'}\right)^2\right) + \\ & i_5^2 m_5 \left(0.25\left(q_1^{'}\right)^2 + 0.25\left(q_2^{'}\right)^2 + 0.25\left(q_4^{'}\right)^2 + 0.25\left(q_5^{'}\right)^2\right) \end{aligned}$$

Determine the potential energy of the links by the matrix formula:

$$P_i = -m_i G^T A_i R_i, \qquad (2)$$

where $R_i = \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}^T$ – matrix column coordinates of the center of gravity link,

$G_i^T = \begin{bmatrix} 0 & 0 & -g & 0 \end{bmatrix}$ – matrix line of gravitational acceleration.

The total potential energy is:

$$P = g \begin{pmatrix} a_1 \left(m_1 + m_2 + m_3 + m_4 + m_5\right) + \\ m_3 q_3 + \left(m_4 + m_5\right)\left(a_4 + q_3\right) - Sin[q_5] m_5 z_5 \end{pmatrix}$$

Create a system of dynamic equations of motion of an industrial robot using the Lagrange equation in matrix form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q'_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial P}{\partial q_i} - Q_i = 0, \qquad (3)$$

where Q_i – generalized forces generated by the electric drive link.

Substituting the kinetic, potential energy and generalized forces into the matrix Lagrange equations, we obtain the system of equations of motion of an industrial robot with five degrees of freedom.

$$q_1^{'}\left(-2Sin[q_2 + q_4] a_2 a_5 m_5 \left(q_2^{'} + q_4^{'}\right)\right) + 0.005 m_1 q_1^{''} +$$

$$\begin{pmatrix} \left(0.005 + a_2^2\right)\left(m_2 + m_3 + m_4\right) + \\ \left(0.005 + a_2^2 + a_5^2 + 2Cos[q_2 + q_4] a_2 a_5\right) m_5 \end{pmatrix} q_1^{''} = Q_1,$$

$$a_5^2 m_5 q_2^{''} + Sin[q_2 + q_4] a_2 a_5 m_5 q_1^{'2} +$$
$$0.005\left(m_2 + m_3 + m_4 + m_5\right) q_2^{''} = Q_2,$$

$$\left(m_3 + m_4 + m_5\right)\left(g + q_3^{''}\right) = Q_3,$$

$$0.005 m_4 q_4^{''} + m_5 \left(Sin[q_2 + q_4] a_2 a_5 q_1^{'2} + \left(0.005 + a_5^2\right) q_4^{''}\right) = Q_4,$$

$$0.005 m_5 q_5^{''} = Q_5, \qquad (4)$$

Integrating the third and fifth equation of the system:

$$q_3(t)=\frac{1}{2}t^2\left(\frac{Q_3}{m_3+m_4+m_5}-g\right);\ q_5(t)=\frac{100t^2Q_5}{m_5};$$

To solve the remaining three differential equations of system (4), we apply the method of polynomial transformations [8, 9] with the following parameters:

$$m_1=200\,kg, m_2=60\,kg, m_3=30\,kg, m_4=20\,kg,$$

$$Q_1=6000, Q_2=1000, Q_3=700, Q_4=900, Q_5=0.01,$$

$$m_5=20\,kg, a_1=30\,cm, a_2=20\,cm,$$

$$a_4=30\,cm, a_5=20\,cm\,.$$

In the calculations, the values for the masses and lengths of the robot links are obtained from the technical characteristics for industrial manipulators.

The transformation method allows us to obtain a solution to a system of differential equations using special polynomial transformations. A large number of papers [11, 12] are devoted to the method of polynomial transformations. The analytical method of transformations allows you to convert the system of differential equations understudy to an autonomous form. The method of transformations allows you to build a solution with all the nonlinear components of the source system. The solution of the system of three differential equations (4) was obtained by the method of transformations:

$$q_1(t)=-0.254272\,Cos[0.175\,t]+0.180296\,Cos[1.128\,t]+ \\ 1.48213\,Sin[0.175\,t]-0.0133503\,Sin[1.128\,t],$$

$$q_2(t)=-0.247989\,Cos[0.223\,t]+0.0348912\,Cos[1.83\,t]+ \\ 1.28984\,Sin[0.223\,t]+0.090897\,Sin[1.83\,t],$$

$$q_4(t)=-0.207264\,Cos[0.196\,t]+0.0341508\,Cos[1.833\,t]+ \\ 1.27545\,Sin[0.196\,t]+0.0722937\,Sin[1.833\,t]$$

$$q_3(t)=0.095t^2,\ q_5(t)=0.05\,t^2.$$

The correctness of the presented mathematical model of the robot is confirmed by the use of the universally recognized matrix method in the kinematics of robots, using the traditional Lagrange matrix equations in the dynamics of robots and verification of calculations in computer mathematical packages.

Fig. 2 shows the calculation of the generalized coordinate $q_1(t)$ for the robot.

Fig. 3 shows the calculation of the generalized coordinate $q_2(t)$ for the robot.

Fig. 4 shows the calculation of the generalized coordinate $q_3(t)$ for the robot.

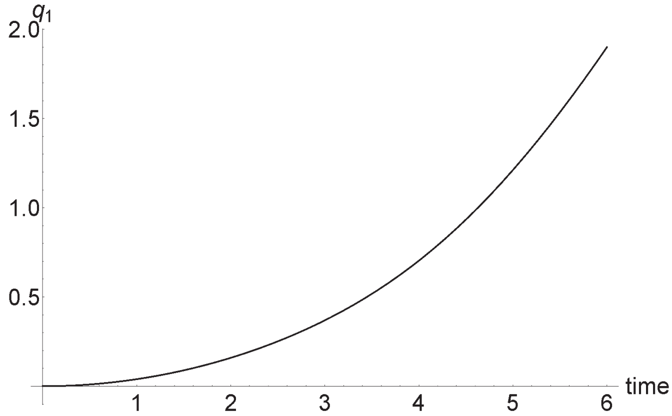

Fig. 2. Generalized coordinate $q_1(t)$ of the link of the robot

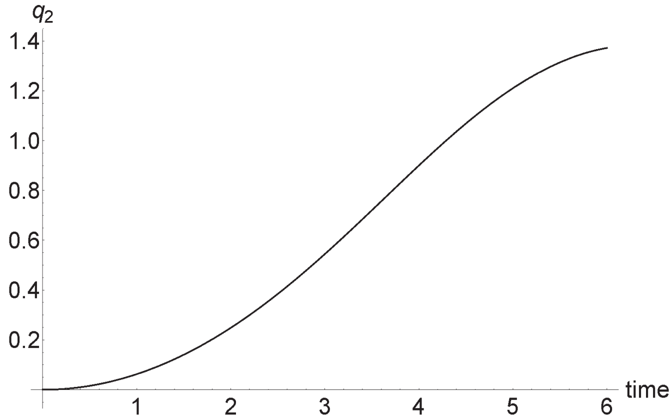

Fig. 3. Generalized coordinate $q_2(t)$ of the link of the robot

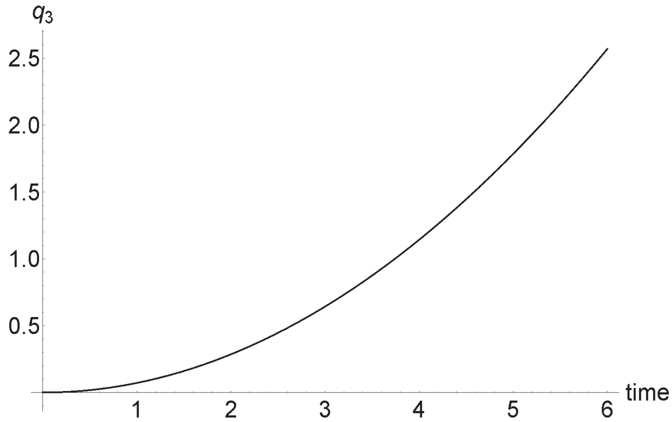

Fig. 4. Generalized coordinate $q_3(t)$ of the link of the robot

Fig. 5 shows the calculation of the generalized coordinate $q_4(t)$ for the robot.

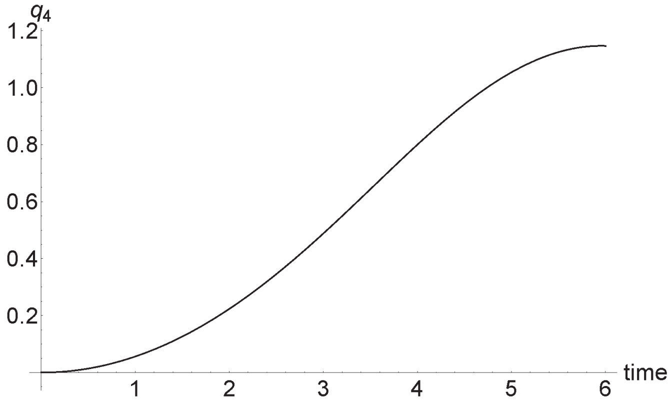

Fig. 5. Generalized coordinate $q_4(t)$ of the link of the robot

Fig. 6 shows the calculation of the generalized coordinate $q_5(t)$ for the robot.

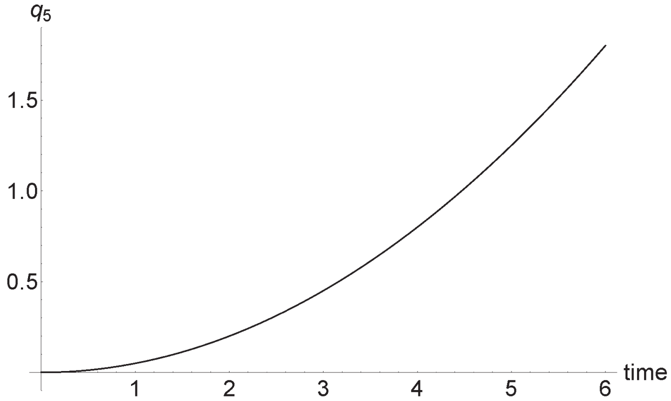

Fig. 6. Generalized coordinate $q_5(t)$ of the link of the robot

Fig. 7 and 8 show the calculation of the generalized velocity q' and acceleration q'' for link q_1 of the robot.

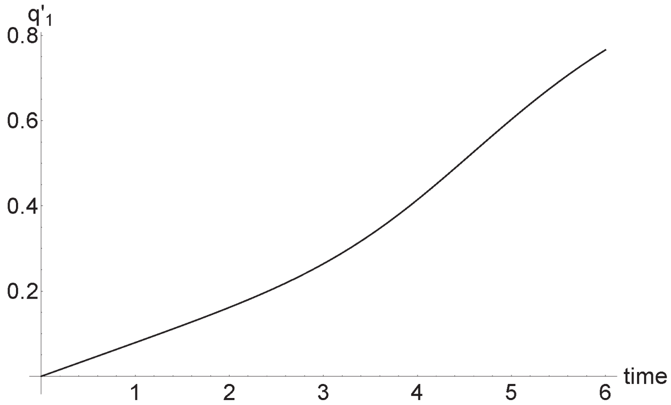

Fig. 7. Generalized speed $q_1'(t)$ of robot link

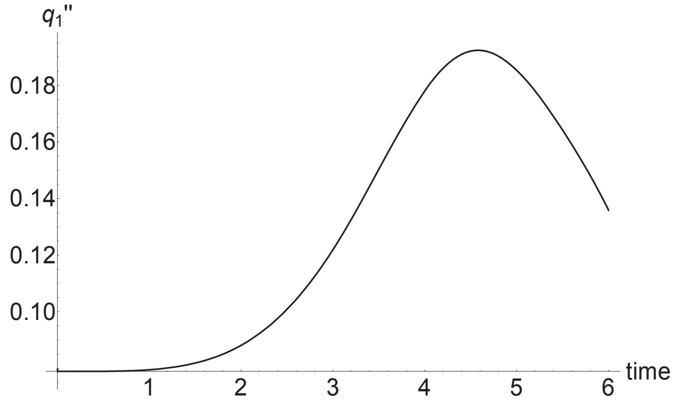

Fig. 8. Generalized acceleration $q_1''(t)$ of robot link

Fig. 9 and 10 show the calculation of the generalized velocity q' and acceleration q'' for link q_2 of the robot.

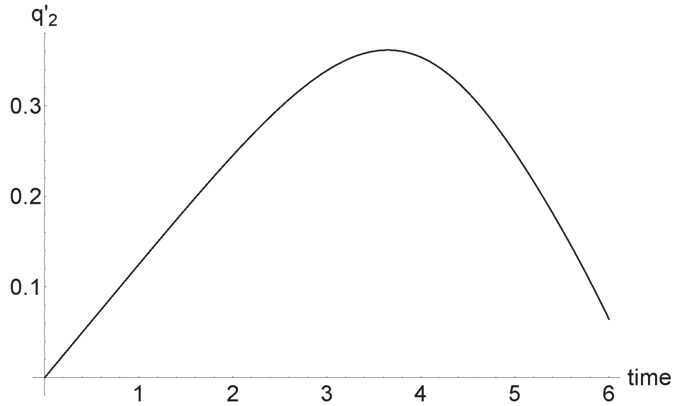

Fig. 9. Generalized speed $q_2'(t)$ of robot link

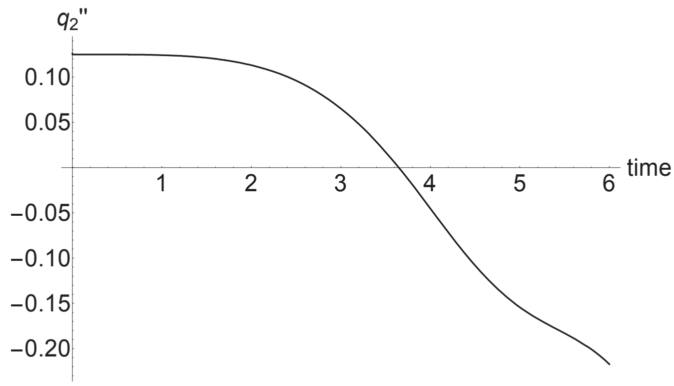

Fig. 10. Generalized acceleration $q_2''(t)$ of robot link

Fig. 11, 12 and 13 show the spatial trajectory of the gripper of the robot in the coordinate systems *xy*, *xz* and *yz*.

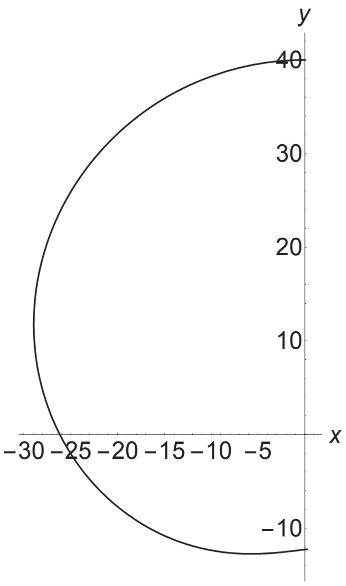

Fig. 11. Trajectory of the gripper in the coordinate system *xy*

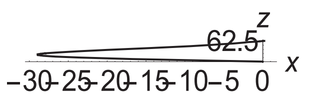

Fig. 12. Trajectory of the gripper in the coordinate system *xz*

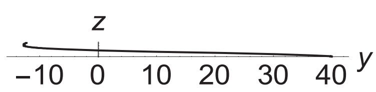

Fig. 13. Trajectory of the gripper in the coordinate system *yz*

Figure 14 shows the spatial trajectory of the gripper of the robot relative to the fixed base of the rack in the initial coordinate system O_0.

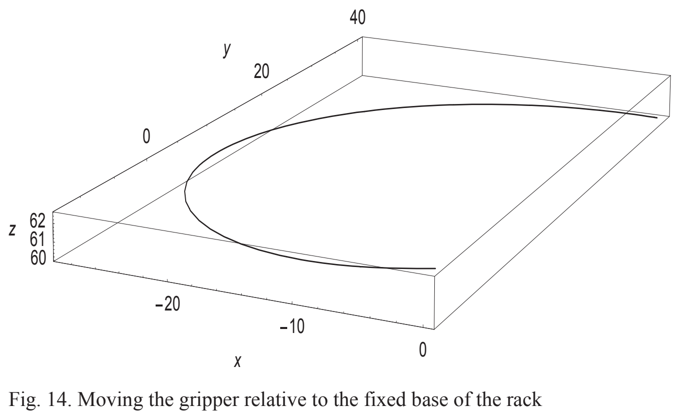

Fig. 14. Moving the gripper relative to the fixed base of the rack

As a result of the calculations of the robot model, we obtain a spatial trajectory of the gripper of the robot, which coincides with the actual path for industrial robotic manipulators MP. This confirms the reliability of the calculations and the results of the analysis of the robot model.

III. Conclusion

The mathematical model of an industrial robot with five degrees of freedom has been obtained and studied. The kinematic scheme of the robot has been determined and the development of a mathematical model has been carried out using the matrix method and the Lagrange matrix dynamic equations. The study of the model was performed using the method of polynomial transformations. As a result of the analysis of the mathematical model, generalized coordinates, velocities and accelerations were obtained for the links of the robot. The spatial trajectory of the movement of the robot's gripper relative to the fixed base of the stand in the initial coordinate system was presented. The developed mathematical model of the robot allows you to determine the main dynamic characteristics of the robot, as well as the necessary generalized forces created by the drive links to move the robot's gripper to the selected point. Also, the novelty of the work lies in the fact that when calculating the robot model, a computer program developed and registered by the authors was used. To control the correctness of the results obtained, the authors conducted a simulation in computer engineering packages, which showed the consistency of the results with theoretical predictions and experimental data. This model allows the development of automated robotic systems and can be used in robot control systems with five degrees of freedom. Based on the presented approach, the authors plan to further develop and investigate models of robots with six degrees of freedom.

References

[1] M. Begnini, D. W. Bertol, N. A. Martins, “A robust adaptive fuzzy variable structure tracking control for the wheeled mobile robot: Simulation and experimental results”, *Control Engineering Practice*, 2017, 64, pp. 27-43.

[2] K. Hauser, “Robust contact generation for robot simulation with unstructured meshes” *Robotics Research*, Springer, Cham, 2016, pp. 357-373.

[3] M. H. Sangdani, A. R. Tavakolpour-Saleh, and A. Lotfavar, “Genetic algorithm-based optimal computed torque control of a vision-based tracker robot: Simulation and experiment”, *Engineering Applications of Artificial Intelligence*, 67, 2018, pp. 24-38.

[4] L.J. Lonnie, “Robot Simulation”, *Robotics and Automation Handbook*, CRC Press, 2018, pp. 411-437.

[5] A. K. Bedaka, C. Y. Lin, “CAD-based robot path planning and simulation using OPEN CASCADE”, *Procedia computer science*, 133, 2018, pp. 779-785.

[6] L. Kunze, M. Beetz, “Envisioning the qualitative effects of robot manipulation actions using simulation-based projections”, *Artificial Intelligence*, 247, 2017, pp. 352-380.

[7] M. Rahmani, A. Ghanbari, M.M. Ettefagh, “Robust adaptive control of a bio-inspired robot manipulator using bat algorithm”, *Expert Systems with Applications*, 56, 2016, pp. 164-176.

[8] Y. Kali, M. Saad, J.P. Kenné, K. Benjelloun, “Robot manipulator control using backstepping with Lagrange’s extrapolation and PI compensator”, *New Developments and Advances in Robot Control*, Springer, Singapore, 2019, pp. 137-161.

[9] T. V. Zudilova, S. E. Ivanov, L. N. Ivanova, “The automation of electromechanical lift for disabled people with control from a mobile device”, *Computing Conference,* IEEE, 2017, pp. 668-674.

[10] E. I. Vorobyov, S. A. Popov, G. I. Sheveleva, *Mechanics of industrial robots: in 3 books, Part 1: Kinematics and dynamics,* M.: Higher. school, 1988, p. 304.

[11] G.I. Melnikov, N.A. Dudarenko, K.S. Malykh, L.N. Ivanova, V.G. Melnikov, “Mathematical models of nonlinear oscillations of mechanical systems with several degrees of freedom”, *Nonlinear Dynamics and Systems Theory*, 17(4), 2017, pp. 369-375.

[12] G. I. Melnikov, S. E. Ivanov, V. G. Melnikov, “The modified Poincare-Dulac method in analysis of autooscillations of nonlinear mechanical systems”, *Journal of Physics*: Conference Series, 570(2), IOP Publishing, 2014, p.022002.