ToA-based Algorithm of Joint Filtering of Coordinates And Time of Transmission for MLAT systems

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***Abstract*—The accuracy of radio radiation source (RRS) coordinates determination in MLAT systems is largely determined by the algorithm used. Traditionally, the MPS use one-step estimates of RRS coordinates obtained by the least squares method (LSM), where differences in measurements of RRS signal reception moments at different receiving stations (RS) are used as input data. This approach ignores some of the information contained in the observations – the moments and period of radio signal emission from RRS. The algorithm of joint filtration of RRS coordinates and time of transmission (ToT) for sampling of sequential observations is presented. The results of modeling of different algorithms for determination of RRS coordinates are presented, and the advantages of the proposed algorithm in comparison with the traditional algorithm based on the TDoA method are shown. Estimation of ToT and its filtration at a known period allows you to get closer to the accuracy provided by Time of Flight (ToF) method, which can not be implemented directly for MLAT systems. This hypothesis was confirmed by modeling with the use of two types of trajectories - circular movement and straight-line movement.**

I. INTRODUCTION

Multilateration (MLAT) systems are widely used in radiolocation and radio control systems. Due to the joint processing of information in the system, obtained at different points of space, the main advantages of MLAT are achieved [1]. The most of such systems use the time difference of arrival (TDoA) method for radio radiation source (RRS) coordinate determination. Moments of radio signal reception at different points are used [2-5]. Most often, the least squares method (LSM) is used for one-step processing of differences in the reception moments of RRS signals [3],[4].

The feature of the MLAT systems is the need to synchronize the time scales of receiving stations (RS). For this purpose, highly stable reference generators synchronized with signals from global satellite navigation systems are used. This approach provides synchronization of the time scales of receiving stations with an error of less than 30 ns [5].

It is known that for TDoA-based MLAT systems Dilution of Precision (DOP) has a great influence [6]. DOP distribution for such systems is characterized by the presence of blind spots [7]. In practice, the configuration of the system is chosen so that RRS are in the outer zone and inevitably fall into the blind zones. This leads to large coordinate errors. A suitable solution is to use other methods, such as ToF (Time of Flight), for which DOP distribution has no blind zones. But ToF measurements are not achievable in MLAT systems because the time of transmission (ToT) of RRS is unknown. If observations are presented as ToA rather than TDoA, it is possible to estimate ToT, which in turn can be smoothed and extrapolated. This approach allows us to get close to the accuracy of ToF methods in MLAT systems, although it is impossible to do it directly.

The main aim of the work is to analyze the potential capabilities of the filtering localization algorithm for MLAT system using ToA measurements and ToT estimation.

II. PROBLEM STATEMENT

Let us assume an MLAT system with configuration in Fig. 1. It is needed to estimate 2D (in horizon plane) RRS coordinates in the topocentric coordinate system associated with MLAT system, provided that the RRS height h is known. Receiving stations positions are $\mathbf{x}_i = (x_i \quad y_i \quad z_i)^T$.

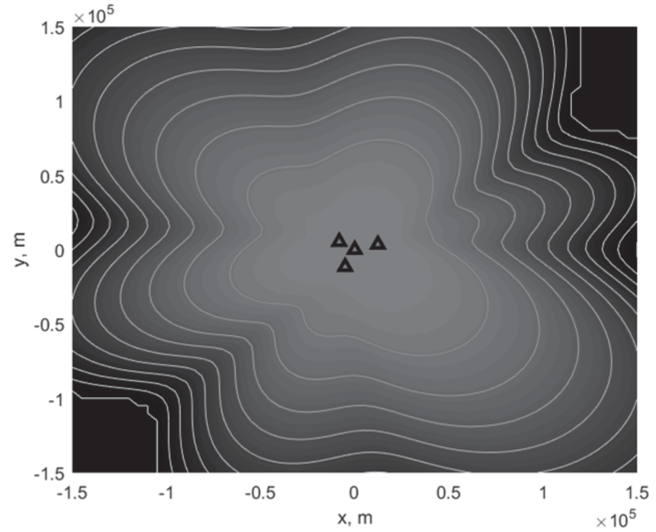

Fig. 1. MLAT system configuration with DOP distribution in the background

Observations are Times of Arrival (ToAs) for each RS. In the classical TDoA setting [4-7], the time differences of reception are formed from these estimates, which are the input data for coordinates calculation algorithms, for example, on the basis of LSM.

In this paper, it is proposed to use the initial estimates of ToAs as input data without forming time differences. Provided that the time scales of all RS are synchronized, the moment of RRS ToT in the MLAT system time scale at the k step of observation ToT_k is unknown and is also subject to estimation. The model of observation is as follows:

$$y_{i,k} = \left(ToT_k + \frac{R_{i,k}}{c}\right) + n_{i,k}, \tag{1}$$

where $R_{i,k} = \|\mathbf{x} - \mathbf{x}_i\|$ −geometric distance between i-th RS and RRS, $n_{i,k}$ − discrete white Gaussian noise with zero mean and constant dispersion $\mathrm{M}\{\mathbf{n}_k{}^2\} = \sigma_n^2\mathbf{I}$. This error caused by error of synchronization, multipath and other factors. Standard deviation of this error when modeling all the algorithms is given $\sigma_n = 10$ ns.

Illustration of the observations model (1) is given on figure 2.

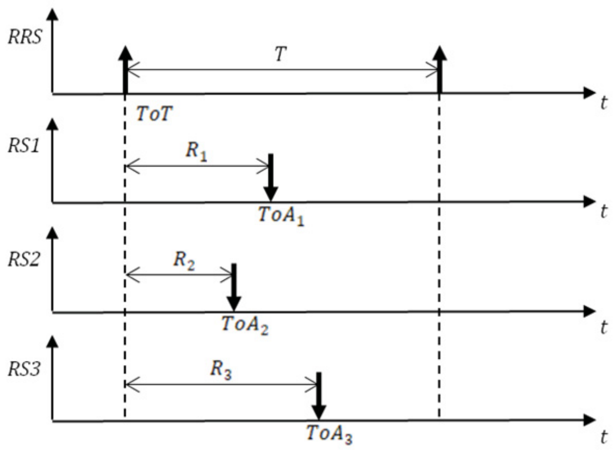

Fig. 2. Observations model (1)

III. DYNAMICS MODEL

It is accepted that RRS emits radio signals with non constant period T_d. In order to achieve the positive effect of transition to the processing of ToA (not TDoA), it is necessary to determine the dynamic model of ToT. From here, the state vector should include the components of the coordinates and velocity of RRS, ToT in MLAT time scale, as well as change of the ToT at the observation step:

$$\boldsymbol{x} = (x \quad V_x \quad y \quad V_y \quad ToT \quad V_\Delta)^T, \tag{2}$$

Dynamics of ToT can be described as follows:

$$ToT_{k+1} = ToT_k + \Delta_k, \tag{3}$$

$$ToT_{k+1} = ToT_k + V_{\Delta,k} \cdot T_d + \xi_{\Delta,k} \cdot T_d, \tag{4}$$

$$\Delta_{i,k+1} - \Delta_{i,k} = \frac{\partial \Delta_{i,k}}{\partial t} \cdot T_d = V_{\Delta,k} \cdot T_d, \tag{5}$$

$$V_{\Delta,k+1} = V_{\Delta,k+1} + \xi_{V\Delta,k} \cdot T_d, \tag{6}$$

Thus, the state vector dynamics model has the following form:

$$\begin{cases} x_{k+1} = x_k + V_{x,k} \cdot T_d \\ V_{x,k+1} = V_{x,k} + \xi_{Vx,k} \cdot T_d \\ y_{k+1} = y_k + V_{y,k} \cdot T_d \\ V_{y,k+1} = V_{y,k} + \xi_{Vy,k} \cdot T_d \\ ToT_{k+1} = ToT_k + V_{\Delta,k} \cdot T_d + \xi_{\Delta,k} \cdot T_d \\ V_{\Delta,k+1} = V_{\Delta,k} + \xi_{V\Delta,k} \cdot T_d \end{cases}, \tag{7}$$

$$\mathbf{F} = \begin{bmatrix} 1 & T_d & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T_d & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T_d \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{8}$$

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ T_d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & T_d & 0 & 0 \\ 0 & 0 & T_d & 0 \\ 0 & 0 & 0 & T_d \end{bmatrix}, \tag{9}$$

where $\boldsymbol{\xi}$ − vector of discrete white Gaussian noises

$$\boldsymbol{\xi} = (\xi_{Vx} \quad \xi_{Vy} \quad \xi_\Delta \quad \xi_{V\Delta})^{\boldsymbol{T}}, \tag{10}$$

with covariance matrix

$$\mathbf{D}_\xi = \begin{bmatrix} \sigma_{Vx}^2 & 0 & 0 & 0 \\ 0 & \sigma_{Vy}^2 & 0 & 0 \\ 0 & 0 & \sigma_\Delta^2 & 0 \\ 0 & 0 & 0 & \sigma_{V\Delta}^2 \end{bmatrix}, \tag{11}$$

IV. ALGORITHM SYNTHESIS

The algorithm of the extended Kalman filter can be one of the variants for solving the problem with the observation model (1) and dynamics model (7-9) of the state vector (2) [10]:

$$\tilde{\mathbf{x}}_k = \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1}, \tag{12}$$

$$\widetilde{\mathbf{D}}_{\mathrm{x},k} = \mathbf{F}_{k-1}\mathbf{D}_{\mathrm{x},k-1}\mathbf{F}_{k-1}^{\mathrm{T}} + \mathbf{G}_{k-1}\mathbf{D}_\xi\mathbf{G}_{k-1}^{\mathrm{T}}, \tag{13}$$

$$\mathbf{K}_k = \widetilde{\mathbf{D}}_{\mathrm{x},k}\frac{\mathrm{d}\mathbf{S}_\mathrm{k}(\tilde{\mathbf{x}}_\mathrm{k})}{\mathrm{d}\mathbf{x}}^{\mathbf{T}}\left(\frac{\mathrm{d}\mathbf{S}_\mathrm{k}(\tilde{\mathbf{x}}_\mathrm{k})}{\mathrm{d}\mathbf{x}}\widetilde{\mathbf{D}}_{\mathrm{x},k}\frac{\mathrm{d}\mathbf{S}_\mathrm{k}(\tilde{\mathbf{x}}_\mathrm{k})}{\mathrm{d}\mathbf{x}}^{-1} + \mathbf{D}_n\right)^{-1}, \tag{14}$$

$$\mathbf{D}_{\mathrm{x},k} = \widetilde{\mathbf{D}}_{\mathrm{x},k} - \mathbf{K}_k\frac{\mathrm{d}\mathbf{S}_\mathrm{k}(\tilde{\mathbf{x}}_\mathrm{k})}{\mathrm{d}\mathbf{x}}\widetilde{\mathbf{D}}_{\mathrm{x},k}, \tag{15}$$

$$\hat{\mathbf{x}}_k = \tilde{\mathbf{x}}_k + \mathbf{K}_k(\mathbf{y}_k - \tilde{\mathbf{y}}_k). \tag{16}$$

According to (1) function **S(x)** is described as follows:

$$\mathbf{S}^i(\mathbf{x}) = ToT + \frac{\sqrt{(x - x_i)^2 + (y - y_i)^2 + (h - z_i)^2}}{c}, \tag{17}$$

or

$$\mathbf{S}^i(\mathbf{x}) = ToT + \frac{\|\mathbf{cx} - \mathbf{x}_i\|}{c}, \tag{18}$$

where $\mathbf{c} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$.

Then

$$\frac{\mathrm{d}\mathbf{S}_{\mathrm{k}}^{i}(\mathbf{x}_{\mathrm{k}})}{\mathrm{d}\mathbf{x}} = \begin{pmatrix} \frac{x-x_i}{\|\mathbf{cx}-\mathbf{x}_i\|} & 0 & \frac{y-y_i}{\|\mathbf{cx}-\mathbf{x}_i\|} & 0 & 1 & 0 \end{pmatrix}. \quad (19)$$

During filtration, the estimation of the time of transmission time is extrapolated to the next time of observation taking into account all received information. In contrast to the period, the time of transmission should have a smaller dispersion value, as it is determined only by direct, not by differential observations. It does not require additional averaging on the observation step, but it is possible to estimate parameters of the model of time of transmission dynamics and use more complex models of this parameter, as well as to modify the filter towards adaptation only by this component of the state vector.

V. DOP COMPARISON

In MLAT systems Dilution of Precision (DOP) value is a measure to reduce the accuracy of coordinate determination due to the peculiarities of the spatial location of RS and RRS. In addition, DOP depends on the used method of determining the coordinates. Further, under the DOP we understand horizontal (HDOP).

DOP is calculated as follows:

$$DOP = \sqrt{trace((\mathbf{H}^T\mathbf{H})^{-1})}. \quad (20)$$

where **H** is a design matrix of used method. Examples of design matrix forms for TDoA, ToA and ToF methods are given in (21)-(23) respectively:

$$\boldsymbol{H_{TDoA}}(\boldsymbol{x}) = \begin{pmatrix} \frac{(\boldsymbol{cx}-\boldsymbol{x}_2)^T}{\|\boldsymbol{cx}-\boldsymbol{x}_2\|} - \frac{(\boldsymbol{cx}-\boldsymbol{x}_1)^T}{\|\boldsymbol{cx}-\boldsymbol{x}_1\|} \\ \frac{(\boldsymbol{cx}-\boldsymbol{x}_3)^T}{\|\boldsymbol{cx}-\boldsymbol{x}_3\|} - \frac{(\boldsymbol{cx}-\boldsymbol{x}_1)^T}{\|\boldsymbol{cx}-\boldsymbol{x}_1\|} \end{pmatrix}, \quad (21)$$

$$\boldsymbol{H_{ToA}}(\boldsymbol{x}) = \begin{pmatrix} \frac{(\boldsymbol{cx}-\boldsymbol{x}_1)^T}{\|\boldsymbol{cx}-\boldsymbol{x}_1\|} & 1 \\ \frac{(\boldsymbol{cx}-\boldsymbol{x}_2)^T}{\|\boldsymbol{cx}-\boldsymbol{x}_2\|} & 1 \\ \frac{(\boldsymbol{cx}-\boldsymbol{x}_3)^T}{\|\boldsymbol{cx}-\boldsymbol{x}_3\|} & 1 \end{pmatrix}, \quad (22)$$

$$\boldsymbol{H_{ToF}}(\boldsymbol{x}) = \begin{pmatrix} \frac{(\boldsymbol{cx}-\boldsymbol{x}_1)^T}{\|\boldsymbol{cx}-\boldsymbol{x}_1\|} \\ \frac{(\boldsymbol{cx}-\boldsymbol{x}_2)^T}{\|\boldsymbol{cx}-\boldsymbol{x}_2\|} \\ \frac{(\boldsymbol{cx}-\boldsymbol{x}_3)^T}{\|\boldsymbol{cx}-\boldsymbol{x}_3\|} \end{pmatrix}. \quad (23)$$

Figure 3 shows the DOP distribution for TDoA and ToA methods, calculated by formula (20) using (21-22) in case of 3 receiving stations. Modeling has shown that DOP for these methods coincides in the whole working area. From this, we can conclude that for the **one-step** coordinate estimation ToA and TDoA are equivalent, as shown in [7].

It may be noted that according to Fig. 3, for ToA and TDoA, a sharp increase of DOP is typical of the exit from the working zone of the MLAT systems (within the boundary of bases between the RSs), as well as the presence of "blind" zones - areas with high DOP value. DOP value at a distance of 100 km reaches several hundred. Figure 4 shows the DOP distribution for the TOF method. TOF method has a much lower value of DOP throughout the working area of the MLAT system as compared to the TDoA and ToA and is characterized by the absence of "blind" zones. Therefore we should strive to apply such algorithms that will allow us to approach the use of TOF in the MLAT systems.

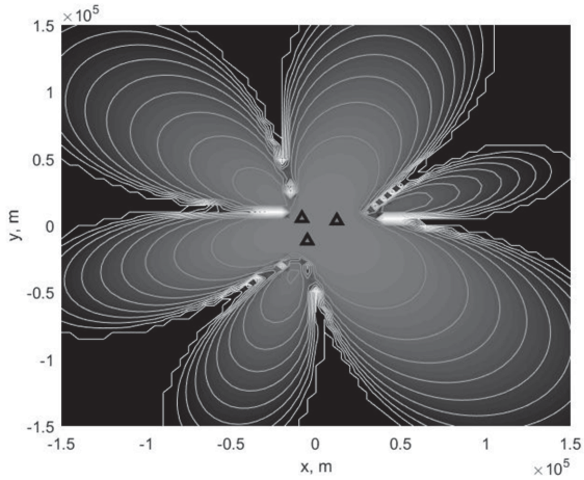

Fig. 3. DOP distribution for ToA/TDoA methods

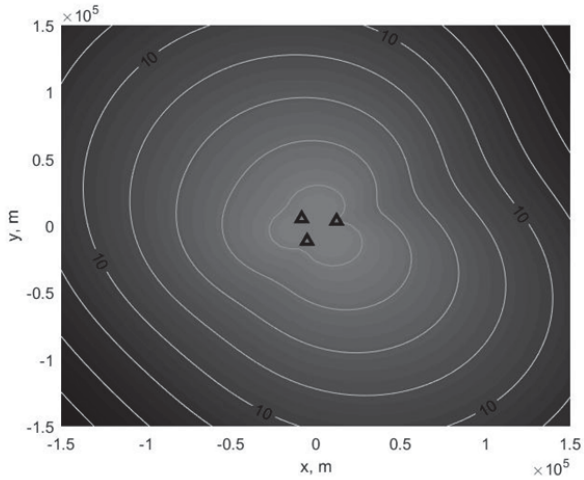

Fig. 4. DOP distribution for ToF method

VI. MODELLING

To estimate the described algorithm efficiency imitation experiment has been held. There were four RS according to Fig. 1 with known coordinates. The RRS trajectory in the working area of the MLAT system was specified. It is accepted that RRS moves at a constant speed of 200 m/s at a known constant height. The RRS flight along the given trajectory and corresponding measurements in the form of ToA according to (1) were simulated. The period of radiation was constant and equal to 1 sec. True values of the state vector (coordinates and ToT) are known. Standard deviation of measurement noise was 10 ns. Three different algorithms were used to process simulation observations and estimate the state vector: traditional TDoA algorithms based on LSM (TDoA LSM), provided filtering ToA algorithm EKF based (ToA Filter), ToF algorithm based on LSM (ToF LSM). As mentioned earlier, the ToF algorithm cannot be used for MLAT system, because ToT is unknown. But in our case we artificially form ToF measurements as $\mathrm{ToF} = \mathrm{ToT} + \mathrm{ToA}$ in order to use ToF state vector estimation as a reference to which the ToA Filter algorithm tends. For all three algorithms, the coordinate estimation errors in the x and y axes and the ToT axes are calculated. or ToF LSM the ToT value is not evaluated. The standard deviation was then calculated.

Two types of trajectories were considered:

a. Flight at a constant speed in the far zone of the MLAT system on a circle radius of 100 km

b. Direct flight at a constant speed

Figures 5-7 show the trajectory estimates for first case for TDoA LSM, ToA Filter and ToF LSM respectively. We can see that the estimation obtained with TDoA LSM is the most noisy.

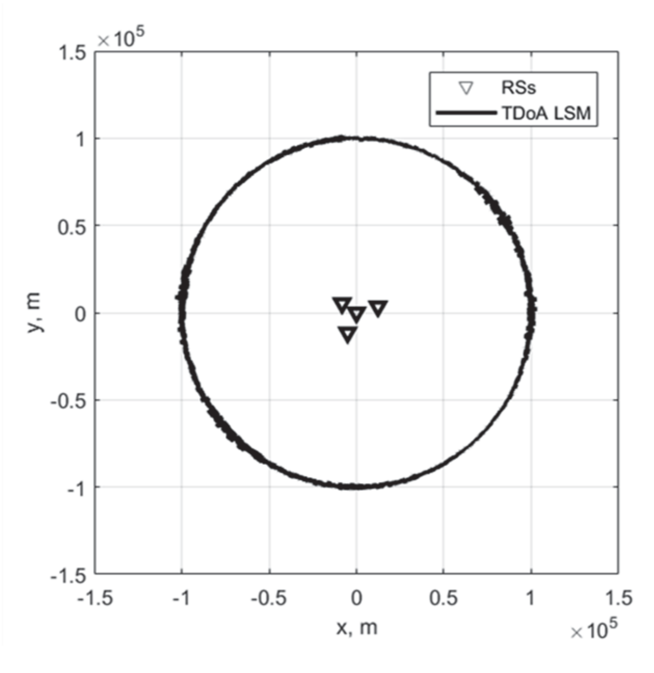

Fig. 5. Circle trajectory estimation using TDoA LSM

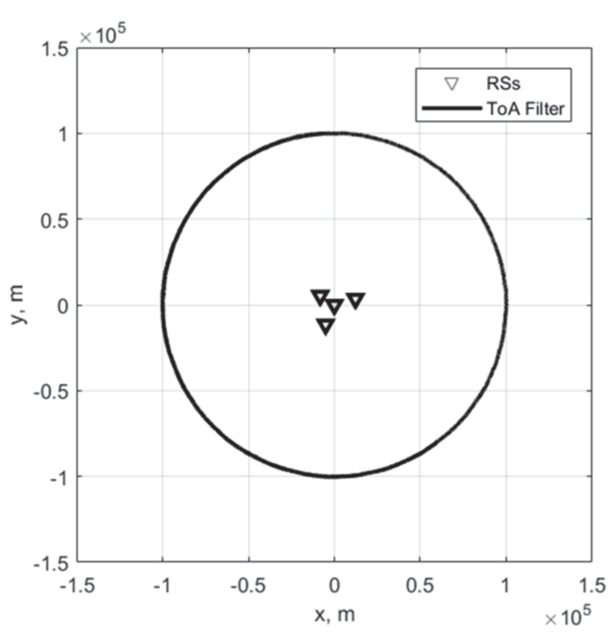

Fig. 6. Circle trajectory estimation using ToA Filter

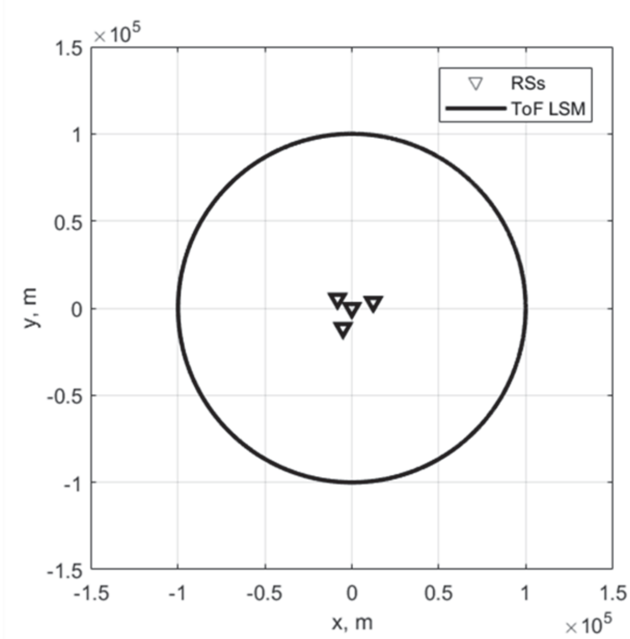

Fig. 7. Circle trajectory estimation using ToF LSM

The coordinate estimation errors for the three methods are shown in Fig. 8, 9. For TDoA LSM, the standard deviation errors in the x and y axes were **634 m** and **536 m** respectively. For ToA Filter method **106 m** and **104 m**, for ToF LSM – **14 m** and **15 m**.

ToT estimation errors for TDoA LSM and ToA Filter are shown in Fig. 10. The standard deviation for TDoA LSM was **2.8 microseconds**, for ToA Filter it was **0.4 microseconds**.

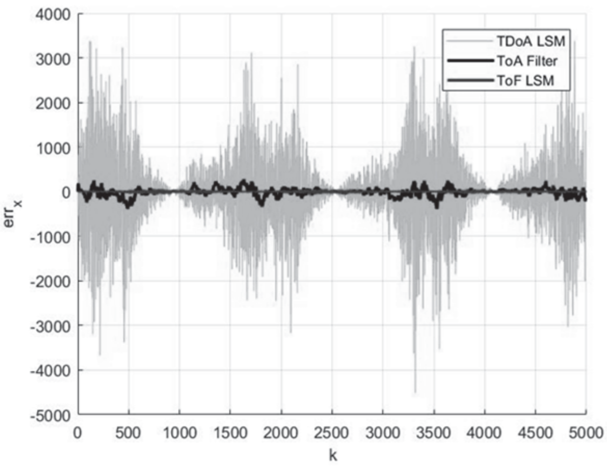

Fig. 8. Circle trajectory x-axis estimation error comparison

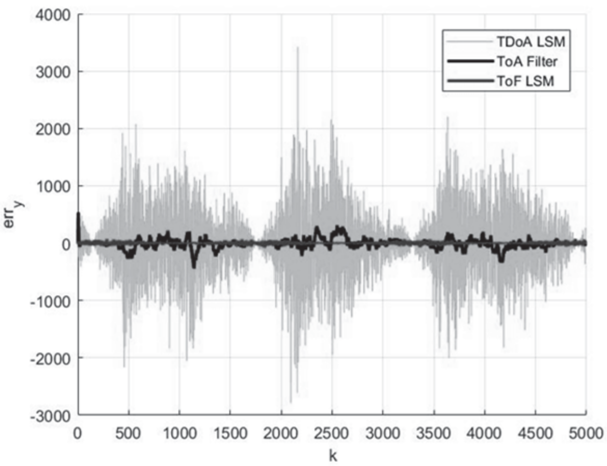

Fig. 9. Circle trajectory y-axis estimation error comparison

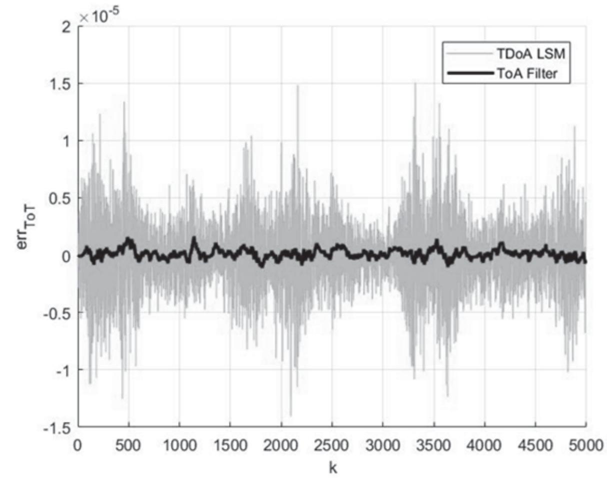

Fig. 10. Circle trajectory ToT estimation error comparison

Fig. 11-13 show the trajectory estimates for second case (direct trajectory) for TDoA LSM, ToA Filter and ToF LSM respectively.

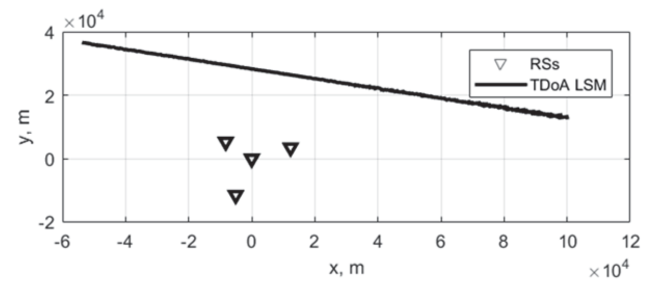

Fig. 11. Direct trajectory estimation using TDoA LSM

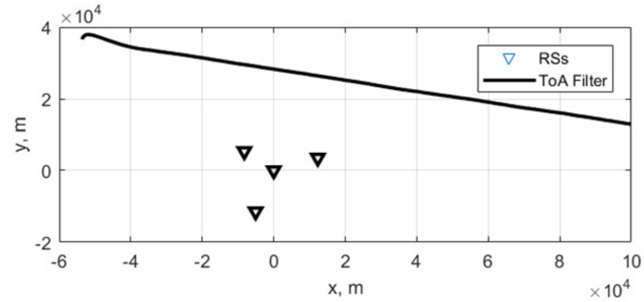

Fig. 12. Direct trajectory estimation using ToA Filter

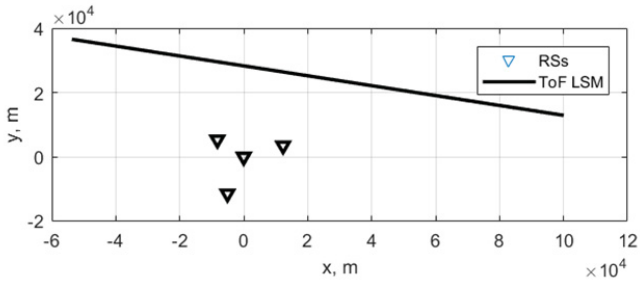

Fig. 13. Direct trajectory estimation using ToF LSM

The coordinate estimation errors for the three methods are shown in Fig. 14, 15. For TDoA LSM, the standard deviation errors in the x and y axes were **375 m** and **115 m** respectively. For ToA Filter method **119 m** and **54 m**, for ToF LSM – **6 m** and **12 m**.

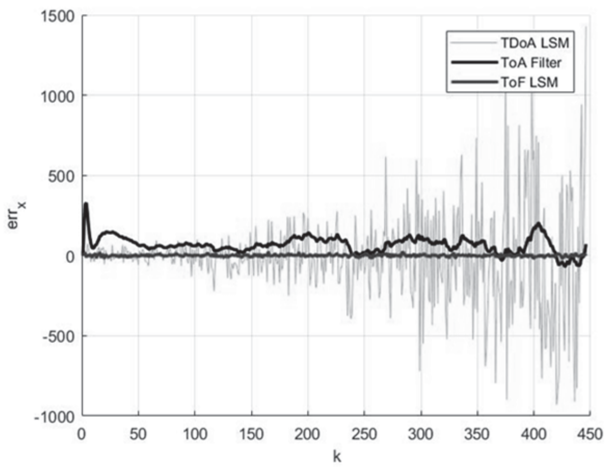

Fig. 14. Circle trajectory x-axis estimation error comparison

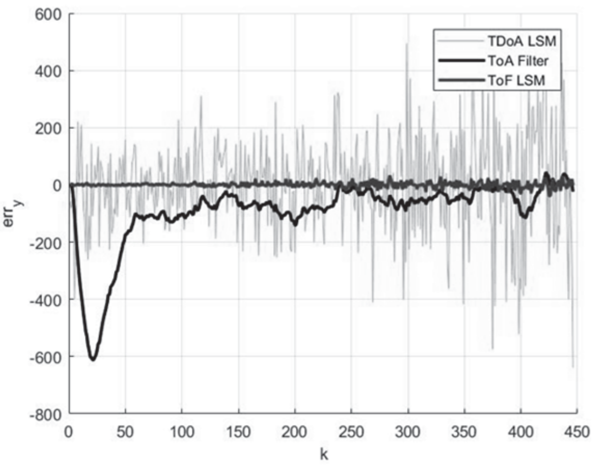

Fig. 15. Direct trajectory y-axis estimation error comparison

ToT estimation errors for TDoA LSM and ToA Filter are shown in Figure 16. The standard deviation for TDoA LSM was **2.8 microseconds**, for ToA Filter it was **0.4 microseconds**.

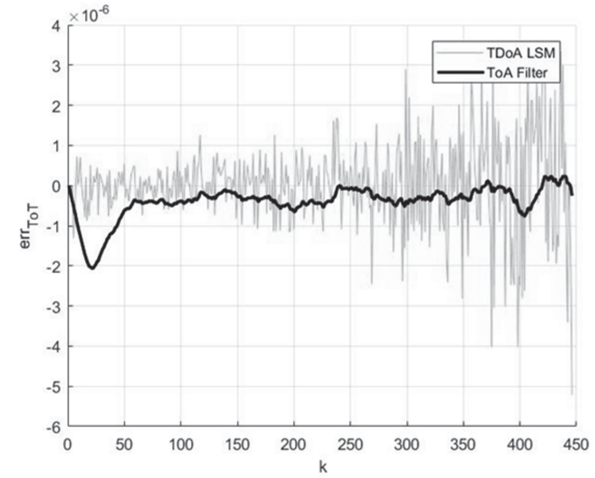

Fig. 16. Direct trajectory ToT estimation error comparison

All modeling results are presented in Table I.

TABLE I. RESULTS OF MODELLING

Trajectory	STD	TDoA LSM	ToA Filter	ToF LSM
Circle	x, m	634	106	14
	y, m	536	104	15
	ToT, sec	$2.8{\cdot}10^{-6}$	$4.4{\cdot}10^{-7}$	-
Direct	x, m	375	119	6
	y, m	115	54	12
	ToT, sec	$1.3{\cdot}10^{-6}$	$3.5{\cdot}10^{-7}$	-

VII. CONCLUSION

In this article, an algorithm for determining RRS coordinates on the basis of EKF with the processing of ToA type measurements is proposed.

The feature of the proposed algorithm is to determine the RRS's ToT in the MLAT system time scale and the period of transmission.

Evaluation of ToT and its filtration at a known period allows you to get closer to the accuracy provided by ToF method, which can not be implemented directly for MLAT systems. This hypothesis was confirmed by modeling with the use of two types of trajectories - circular movement and straight-line movement.

Future work is to implement the provided approach and determine its effectiveness in case of non-constant period of transmission.

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