# Quasi-Orthogonal Space-Time Block Coding with Closed-Loop Control in MIMO Communication Systems 

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#### Abstract

In the paper, we proposed a method of space-time diversity on the transmitting side, combining a closed feedback and quasi-orthogonal space-time coding technique called closedloop control with quasi-orthogonal space-time block coding (QOSTBC). For a configuration $4 \times 1-4 \times 4$ with four transmit and one receive antennas and four symbols per four resource elements, our method allows the QO-STBC to be completely orthogonalized what maximizes the diversity gain. It is shown that the developed approach can also be used for the $4 \times 2-4 \times 2$ configuration. In this case, it is not possible to orthogonalize QO-STBC completely, but the level of inter-channel correlation is considerably reduced. A statistical modeling was carried out to study the characteristics of noise immunity of the proposed transmission scheme. We show that it is sufficient to use a one-bit command to control the phase of two transmitting antennas. A gain of 1.4 dB is achieved for the $4 \times 1-4 \times 4$ scenario and 2.5 dB - for the $\mathbf{4 \times 2}-4 \times 2$ case. Phase control of transmitting antennas used in this method of signal transmission does not change the interference conditions.


## I. Introduction

In [1], a Space-Time Transmit Diversity (STTD) algorithm was proposed for two transmit antennas. This algorithm used Orthogonal Space-Time Block Coding at a coding rate of 1 and allowed maximizing the diversity gain from spacial distribution of two transmit antennas.

Subsequently, numerous attempts were made to find similar codes for a larger number of antennas, e.g. for four transmitting antennas and more. In the end, the impossibility of the existence of such codes with a rate of 1 for 4 transmitting antennas was proved in [2]. However, similar codes have been found with coding rates $3 / 4$ and $1 / 2$ [3]. At the same time, numerous combinations of STTD and Orthogonal Transmit Diversity (OTD) have been investigated in [4], [5], [6]. The proposed approaches are simple for encoding and decoding, but when used alone, they provide a modest diversity gain of two with four transmit and one receive antennas [7], [8].

However, by concatenating such space-time codes with error correcting codes (e.g., convolutional or Turbo codes), the diversity gain is increased. This is due to the fact that when the received signal is processed, two sequences of symbols with independent fading are formed. In this case, the correcting ability of the error-correcting code is used more effectively.

However, the maximum possible diversity gain is achieved only at low coding rates.
In the above listed diversity transmission algorithms, information about the state of the communication channel is not used at the transmitting side, since it is usually not known at there. Such systems are usually referred to as open-loop.

Another approach to use transmit diversity to improve the efficiency of communication systems is to control the transmit antenna weights based on channel information transmitted from the mobile station back to the base station. Such systems are called closed-loop control systems. The effectiveness of such systems largely depends on the amount and accuracy of information transmitted from the mobile station to the base station to control the transmitting antennas.
Therefore, in this paper, we explore the possibility of using the known STTD and closed-loop control circuits together to improve the energy efficiency of cellular communication systems.

The rest of the paper is arranged as follows. In the next section, we define our system model. In Section III, quasiorthogonal space-time bloc codes (QO-STBC) for the $4 \times 1$ MIMO system with closed-loop feedback are derived. In the following Section IV, similar QO-STBC are derived for the $4 \times 2$ system. Section V contains modeling results and their analysis. Finally, we conclude in Section VI.

## II. System model

We will consider a systems with $M$ transmitting and $N$ receiving antennas. We assume that slow nonselective fading takes place in the communication channel. In this case, the model of the received signal is:

$$
\begin{equation*}
Y=H X+\eta \tag{1}
\end{equation*}
$$

where

- $H \in C^{N \times M}$ is a complex channel matrix constructed from arbitrary transmission coefficients $h_{i j}$ between $j$-th transmitting and $i$-th receiving antenna.
- $\eta \in C^{N}$ is $N$-dimensional complex Gaussian vector of observation noise with zero mean and correlation matrix $2 \sigma_{\eta}^{2} I_{N}$, where $I_{N}$ is unit matrix of size $(N \times M)$.
- $X \in C^{M}$ is $M$-dimensional vector $j$-th element of which represents the complex information symbol transmitted by the $j$-th antenna.
- $Y \in C^{M}$ is the vector with elements received by $i$-th antenna.
Such a model is illustrate in Fig. 1. Systems with sequential cascading of error-correcting code and STTD schemes [8], [9], [12], [14], [15] are considered.


Fig. 1. Communication system with $M$ transmitting and $N$ receiving antennas

In the following derivations, we need to pass from a complex-valued model to a real-valued representation. In this case, each complex variable is associated with a twodimensional vector, the product of complex numbers is described as the product of a two-dimensional vector by a matrix of size ( 2 x 2 ).
An example of transition from complex-valued vectors and matrices for the case $M=N=1$ is given below:

$$
\left[\begin{array}{l}
y_{1, r} \\
y_{1, i}
\end{array}\right]=\left[\begin{array}{cc}
h_{11, r} & -h_{11, i} \\
h_{11, i} & h_{11, r}
\end{array}\right]\left[\begin{array}{c}
z_{1, r} \\
z_{1, i}
\end{array}\right]+\left[\begin{array}{l}
\eta_{1, r} \\
\eta_{1, i}
\end{array}\right],
$$

where the subscript $r$ means the real part and the subscript $i$ means the imaginary part of the corresponding complex element. The transition from a complex model to a real one is described in more detail in [16].

## III. QUASI-ORTHOGONAL SPACE-TIME BLOCK CODES FOR THE 4X1 MIMO SYSTEM WITH CLOSED-LOOP FEEDBACK

The following variants of quasi-orthogonal space-time codes with a minimum number of nonzero cross-correlation coefficients are known [7], [10]. These codes are described by the following matrix transformation rules:

$$
\begin{align*}
F^{(1)}(S) & =\left[\begin{array}{cccc}
s_{1} & -s_{2}^{\prime} & -s_{3}^{\prime} & s_{4} \\
s_{2} & s_{1}^{\prime} & -s_{4}^{\prime} & -s_{3} \\
s_{3} & -s_{4}^{\prime} & s_{1}^{\prime} & -s_{2} \\
s_{4} & s_{3}^{\prime} & s_{2}^{\prime} & s_{1}
\end{array}\right],  \tag{2}\\
F^{(2)}(S) & =\left[\begin{array}{cccc}
s_{1} & -s_{3}^{\prime} & -s_{4}^{\prime} & s_{2} \\
s_{2} & s_{4}^{\prime} & s_{3}^{\prime} & s_{1} \\
s_{3} & s_{1}^{\prime} & -s_{2}^{\prime} & -s_{4} \\
s_{4} & -s_{2}^{\prime} & s_{1}^{\prime} & -s_{3}
\end{array}\right], \tag{3}
\end{align*}
$$

$$
F^{(3)}(S)=\left[\begin{array}{cccc}
s_{1} & -s_{4}^{\prime} & -s_{2}^{\prime} & s_{3}  \tag{4}\\
s_{2} & -s_{3}^{\prime} & s_{1}^{\prime} & -s_{4} \\
s_{3} & s_{2}^{\prime} & s_{4}^{\prime} & s_{1} \\
s_{4} & s_{1}^{\prime} & -s_{3}^{\prime} & -s_{2}
\end{array}\right],
$$

where $s_{i}$ are complex QAM symbols.
To analyze further the properties of the reduced STBC matrices, we pass to the real-valued representation. Let us consider this transfer using an example of a matrix of the form (2). In its real-valued form, it can be written as follows:

$$
F^{(1)}(S)=\left[\begin{array}{cccc}
s_{1, r} & -s_{2, r} & -s_{3, r} & s_{4, r} \\
s_{1, i} & s_{2, i} & s_{3, i} & s_{4, i} \\
s_{2, r} & s_{1, r} & -s_{4, r} & -s_{3, r} \\
s_{2, i} & -s_{1, i} & s_{4, i} & -s_{3, i} \\
s_{3, r} & -s_{4, r} & s_{1, r} & -s_{2, r} \\
s_{3, i} & s_{4, i} & -s_{1, i} & -s_{2, i} \\
s_{4, r} & s_{3, r} & s_{2, r} & s_{1, r} \\
s_{4, r} & -s_{3, r} & -s_{2, r} & s_{1, i}
\end{array}\right],
$$

where $s_{k, r}$ and $s_{k, i}$ are real and imaginary components of $k$ th complex QAM symbol.

Generation of this matrix can be described in the following way:

$$
F^{(1)}(S)=\left[\begin{array}{llll}
F_{1}^{(1)} S & F_{2}^{(1)} S & F_{3}^{(1)} S & F_{4}^{(1)} S
\end{array}\right],
$$

where $S=\left[s_{1, r} s_{1, i} \ldots s_{k, r} s_{k, i} \ldots s_{4, r} s_{4, i}\right]$ is 8-dimensional vector of information symbols, and $F_{1}^{(1)}, F_{2}^{(1)}, F_{3}^{(1)}, F_{4}^{(1)}$ are generating matrices of the following form:

$$
\begin{aligned}
F_{1}^{(1)} & =\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right], \\
F_{2}^{(1)} & =\left[\begin{array}{cccccccc}
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0
\end{array}\right], \\
F_{3}^{(1)} & =\left[\begin{array}{cccccccc}
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0
\end{array}\right],
\end{aligned}
$$

$$
F_{3}^{(1)}=\left[\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] .
$$

Using matrices (2) - (4), the model (1) for the channel with configuration ( $4 \times 1$ ) takes the form:

$$
Y=h F(S)+\eta
$$

where $Y$ is a four-dimensional complex row-vector of observations, with elements equal to the samples obtained at one time interval; $h$ is a four-dimensional complex channel row-vector; $F(S)$ - STBC matrix with size $(4 \times 4)$, which is described by one of expressions (2) - (4); $S$ - four-dimensional complex vector of transmitted QAM symbols; $\eta$ is a four-dimensional row-vector of complex noise.
Let us pass to the real-valued notations and consider the model for one time interval:

$$
y_{k}=\left[\begin{array}{l}
y_{k, r} \\
y_{k, i}
\end{array}\right]=h F_{k}^{(1)} S+\eta_{k}, k=1, \ldots, 4,
$$

where

$$
\left[\left[\begin{array}{cc}
h_{11, r} & -h_{11, i} \\
h_{11, i} & h_{11, r}
\end{array}\right] \cdots\left[\begin{array}{cc}
h_{41, r} & -h_{41, i} \\
h_{41, i} & h_{41, r}
\end{array}\right]\right]
$$

is a real-valued channel matrix with configuration (4x1) and of shape ( 2 x 8 ). Taking last equation into account we can introduce an equivalent channel matrix for the $k$ th time interval:

$$
h_{k}=h F_{k}^{(1)}
$$

By joining two-dimensional real-valued observed vectors into one vector, it becomes possible to formulate the following model:

$$
Y=\left[\begin{array}{l}
y_{1} \\
y_{2} \\
y_{3} \\
y_{4}
\end{array}\right]=\left[\begin{array}{l}
h F_{1}^{(1)} \\
h F_{2}^{(1)} \\
h F_{3}^{(1)} \\
h F_{4}^{(1)}
\end{array}\right] S+\eta .
$$

The matrix

$$
\tilde{H}^{(1)}=\left[\begin{array}{l}
h F_{1}^{(1)} \\
h F_{2}^{(1)} \\
h F_{3}^{(1)} \\
h F_{4}^{(1)}
\end{array}\right]
$$

is the equivalent channel matrix for the STBC code generated by a matrix of the form (2). The properties of the STBC code will be determined by the matrix

$$
R^{(1)}=\left(\tilde{H}^{(1)}\right)^{T} \tilde{H}^{(1)}
$$

which is, in fact, a correlation matrix for the columns of the equivalent channel matrix. For an orthogonal STBC code, this matrix will be diagonal.

Space-time codes (2), (3) and (4) correspond to the following correlation matrices of the equivalent virtual channel:

$$
\begin{align*}
& R^{(1)}=\left[\begin{array}{cccccccc}
d & 0 & 0 & 0 & 0 & 0 & -2 a & 0 \\
0 & d & 0 & 0 & 0 & 0 & 0 & -2 a \\
0 & 0 & d & 0 & 2 a & 0 & 0 & 0 \\
0 & 0 & 0 & d & 0 & 2 a & 0 & 0 \\
0 & 0 & 2 a & 0 & d & 0 & 0 & 0 \\
0 & 0 & 0 & 2 a & 0 & d & 0 & 0 \\
-2 a & 0 & 0 & 0 & 0 & 0 & d & 0 \\
0 & -2 a & 0 & 0 & 0 & 0 & 0 & d
\end{array}\right],  \tag{5}\\
& R^{(2)}
\end{aligned} \begin{aligned}
& \text { (1) } \tag{6}
\end{align*}
$$

where

$$
\begin{gathered}
d=\sum_{m=1}^{M}\left|h_{m}\right|^{2}, \\
a=\Re\left\{h_{2} h_{3}^{\prime}-h_{1} h_{4}^{\prime}\right\}, \\
b=\Re\left\{h_{1} h_{2}^{\prime}-h_{3} h_{4}^{\prime}\right\}, \\
c=\Re\left\{h_{1} h_{3}^{\prime}-h_{2} h_{4}^{\prime}\right\},
\end{gathered}
$$

It can be seen that the correlation coefficients for each of the given matrices are determined by one value. So, for example, for the space-time code (2) we have:

$$
|2 a|=2\left|\Re\left\{h_{2} h_{3}^{\prime}-h_{1} h_{4}^{\prime}\right\}\right| .
$$

Let us introduce the following identical turning factor for the 1 st and 2 nd transmitting antennas:

$$
\begin{equation*}
\Theta_{1,2}=\exp \left\{j \theta_{1,2}\right\} \tag{8}
\end{equation*}
$$

Then the value of the correlation coefficient, taking into account the turning factor (8), will be equal to:

$$
|2 a|=2\left|\Re\left\{\Theta_{1,2}\left(h_{2} h_{3}^{\prime}-h_{1} h_{4}^{\prime}\right)\right\}\right| .
$$

If we choose the phase of the turning factor equal to

$$
\begin{equation*}
\theta_{1,2}=\frac{\pi}{2}-\arg \left(h_{1} h_{4}^{\prime}-h_{2} h_{3}^{\prime}\right) \tag{9}
\end{equation*}
$$

then all the correlation coefficients in the correlation matrix $R^{(1)}$ from equation (5) will have zero value. If the number of receiving antennas is greater, and the configuration of the space-time code does not change, then the value of the correcting phase of the turning factor in (8) will be calculated in the same way, using the following formula:

$$
\begin{equation*}
\theta_{1,2}=\frac{\pi}{2}-\arg \left(\sum_{n=1}^{N} h_{1, n} h_{4, n}^{\prime}-h_{2, n} h_{3, n}^{\prime}\right) \tag{10}
\end{equation*}
$$

Thus, by adjusting the phases of the first and second transmitting antennas by the same value $\theta_{1,2}$, the correlation matrix $R^{(1)}$ from (5) corresponding to the space-time code can be orthogonalized. In this case, the diversity order will have a maximum value of 4 (with one receive antenna).
Similarly, for the second version of the space-time code (3), we have:

$$
|2 b|=2\left|\Re\left\{h_{1} h_{2}^{\prime}-h_{3} h_{4}^{\prime}\right\}\right| .
$$

For this option, it is necessary to adjust the phases of the 1 st and 3rd transmitting antennas to the same angle:

$$
\begin{equation*}
\theta_{1,2}=\frac{\pi}{2}-\arg \left(h_{1} h_{2}^{\prime}-h_{3} h_{4}^{\prime}\right) . \tag{11}
\end{equation*}
$$

For the third version of the space-time code (4) we have:

$$
|2 c|=2\left|\Re\left\{h_{1} h_{3}^{\prime}-h_{2} h_{4}^{\prime}\right\}\right| .
$$

For this option, the phases of the 1st and 4th antennas are adjusted by the value:

$$
\begin{equation*}
\theta_{1,2}=\frac{\pi}{2}-\arg \left(h_{1} h_{3}^{\prime}-h_{2} h_{4}^{\prime}\right) \tag{12}
\end{equation*}
$$

A block diagram of the formation of a diversity signal on the transmitting side, taking into account the turning factor for the first version of the space-time code (2), is shown in Fig. 2.


Fig. 2. Quasi-orthogonal space-time code in a closed loop system
It should be noted that in the proposed version of space-time coding in a closed loop system, all the necessary calculations for adjusting the phase of the antennas are performed at the mobile station. Only one real number is transmitted on the return channel, regardless of the number of receiving antennas. This significantly reduces the amount of transmitted information and imposes less stringent requirements on the throughput of the reverse control channel.
IV. QUASI-ORTHOGONAL SPACE-TIME BLOCK CODES FOR THE 4X2 MIMO SYSTEM WITH CLOSED-LOOP FEEDBACK
Consider the possible options for combining a quasiorthogonal space-time code and a closed loop system with a $4 \times 2$ configuration. With this configuration, it is possible to use Punctured Quasi-Orthogonal STBC, which is formed from a conventional quasi-orthogonal space-time code by puncturing, in this case by discarding the last two columns of the code matrix. So, for example, from a matrix of the form (2) we obtain the following encoding matrix

$$
\widetilde{F}^{(1)}(S)=\left[\begin{array}{cc}
s_{1} & -s_{2}^{\prime}  \tag{13}\\
s_{2} & s_{1}^{\prime} \\
s_{3} & -s_{4}^{\prime} \\
s_{4} & s_{3}^{\prime}
\end{array}\right]
$$

It is easy to see that further puncturing (discarding the last column) will lead to the usual spatial multiplexing scheme with a maximum space-time coding rate of 4 .

The correlation matrix shown in equation (14) corresponds to coding schemes of the form (13). The following notations have been used in (14):

$$
\begin{gathered}
A=\left|h_{1}\right|^{2}, B=\left|h_{2}\right|^{2}, C=\left|h_{3}\right|^{2}, D=\left|h_{4}\right|^{2}, \\
E=h_{1, r} h_{2, r}+h_{1, i} h_{2, i}, F=-h_{1, i} h_{2, r}+h_{1, r} h_{2, i}, \\
G=h_{1, r} h_{3, r}+h_{1, i} h_{3, i}, H=-h_{1, i} h_{4, r}+h_{1, r} h_{4, i}, \\
K=h_{1, r} h_{4, r}+h_{1, i} h_{4, i}, L=-h_{1, i} h_{4, r}+h_{1, r} h_{4, i}, \\
M=h_{2, r} h_{3, r}+h_{2, i} h_{3, i}, N=-h_{2, i} h_{3, r}+h_{2, r} h_{3, i}, \\
Q=h_{2, r} h_{4, r}+h_{2, i} h_{4, i}, P=-h_{2, i} h_{4, r}+h_{2, r} h_{4, i}, \\
R=h_{3, r} h_{4, r}+h_{4, i} h_{3, i}, S=-h_{3, i} h_{4, r}+h_{4, r} h_{3, i} .
\end{gathered}
$$

In these expressions an index $r$ corresponds to the real part and an index $i$ - to the imaginary part of the complex number.

The total power of the correlation peaks (taking into account several receiving antennas) for the correlation matrix (14) will be equal to

$$
\begin{gathered}
\sum|R|^{2}=(G+Q)^{2}+(H-P)^{2}+(M-K)^{2}+(N+L)^{2}= \\
=\left|H_{31}+H_{24}\right|^{2}+\left|H_{32}-H_{14}\right|^{2}
\end{gathered}
$$

where

$$
\begin{aligned}
& H_{31}=\sum_{n=1}^{N} h_{3, n} h_{1, n}^{\prime}, H_{24}=\sum_{n=1}^{N} h_{2, n} h_{4, n}^{\prime}, \\
& H_{32}=\sum_{n=1}^{N} h_{3, n} h_{2, n}^{\prime}, H_{14}=\sum_{n=1}^{N} h_{1, n} h_{4, n}^{\prime} .
\end{aligned}
$$

Let us introduce, as in the previous Section, a turning factor $\Theta_{1,2}=\exp \left\{j \theta_{1,2}\right\}$ common for the first and second antennas. Then the total power of the correlation peaks will depend on the phase of the turning factor as follows:

$$
\begin{gathered}
\sum\left|R\left(\theta_{1,2}\right)\right|^{2}=\left|H_{31} e^{-j \theta_{1,2}}+H_{24} e^{j \theta_{1,2}}\right|^{2}+ \\
+\left|H_{32} e^{-j \theta_{1,2}}-H_{14} e^{j \theta_{1,2}}\right|^{2}
\end{gathered}
$$

$$
R=\left[\begin{array}{cccccccc}
A+B & 0 & 0 & 0 & G+Q & -H+P & K-M & -L-N  \tag{14}\\
0 & A+B & 0 & 0 & H-P & G+Q & L+N & K-N \\
0 & 0 & B+A & 0 & M-K & -N-L & Q+G & -P+H \\
0 & 0 & 0 & B+A & N+L & M-K & P-H & Q+G \\
G+Q & H-P & M-K & N+L & C+D & 0 & 0 & 0 \\
-H+P & G+Q & -N-L & M-K & 0 & C+D & 0 & 0 \\
K-M & L+N & Q+G & P-H & 0 & 0 & D+C & 0 \\
-L-N & K-N & -P+H & Q+G & 0 & 0 & 0 & D+C
\end{array}\right] .
$$

Fig. 3 shows the dependencies of the normalized value

$$
\lambda\left(\theta_{1,2}\right)=\frac{\sum\left|R\left(\theta_{1,2}\right)\right|^{2}}{\sum|R(0)|^{2}}
$$

on the value of the angle $\theta_{1,2}$ of the turning factor for several realizations of the random elements of the channel matrix. It can be seen from them that the dependencies are sinusoidal and have a clear minimum.


Fig. 3. Dependence of the normalized value $\lambda\left(\theta_{1,2}\right)$ on the value of the angle $\theta_{1,2}$ of the turning factor

Thus, by adjusting the phases of the first and second transmit antennas to the same value, it is possible to reduce the total level of the correlation peaks of the space-time matrix for the ( $4 \times 2$ ) configuration. The structural diagram of the system will be the same as in the previous version, except for the use of a truncated coding matrix. The control requires the transmission of one real number on the reverse control channel, regardless of the total number of receiving antennas.
It should be noted that complete suppression of correlation peaks by rotating the phases of the transmitting antennas in this case is impossible. Therefore, the use of additional turning factors for other antennas does not increase the energy efficiency.

## V. Modelling results

In order to study the characteristics of the proposed algorithm, modeling was carried out for channels with slow
independent Rayleigh fading. The simulated system corresponds to the structural diagram shown in Fig. 2, i.e. sequential cascading of error correcting code and quasi-orthogonal spacetime code with phase control of two transmitting antennas is used. The 8 -state Turbocode [11], [13] was used as the errorcorrecting code.


Fig. 4. Characteristics of noise immunity of various variants of a system with quasi-orthogonal space-time coding for the configuration $4 \times 1-4 \times 4$

Fig. 4 shows the dependencies of frame error rate (FER) on energy per bit to noise power spectral density ratio ( $\mathrm{Eb} / \mathrm{No}$ ) for the following options for constructing a communication system with a space-time matrix (2):

- System without feedback [7];
- A closed loop system with a turning factor (9) with a onebit control command (two phase values $\theta_{1,2} \in\left\{0 ; \frac{\pi}{2}\right\}$ );
- A closed loop system with a turning factor (9) with a two-bit control command (four phase values $\theta_{1,2} \in$ $\left\{0 ; \frac{\pi}{4} ; \frac{\pi}{2} ; \frac{3 \pi}{4}\right\}$ );
- A closed loop system with a turning factor (9) with a three-bit control command (eight phase values $\theta_{1,2} \in$ $\left\{0 ; \frac{\pi}{8} ; \frac{\pi}{4} ; \frac{3 \pi}{8} ; \frac{\pi}{2} ; \frac{5 \pi}{8} ; \frac{3 \pi}{4} ; \frac{7 \pi}{8}\right\}$ ).
The frame length is 1277 bits, using QPSK modulation, $\frac{1}{2}$ rate turbo code. The system uses 4 transmit antennas and 1 receive antenna. The MMSE demodulator was used for all variants. Evaluation of complex channel multipliers and transmission of control commands were assumed to be ideal.


Fig. 5. Characteristics of noise immunity of various variants of a system with quasi-orthogonal space-time coding for the configuration $4 \times 2-4 \times 2$

It can be seen from the given dependences that at $\mathrm{FER}=1 \%$, the introduction of only one-bit control command increases the energy efficiency by 1.4 dB in comparison with the system without feedback.

For a one-bit control command, the phase of the turning factor takes only two values $\theta_{1,2} \in\left\{0 ; \frac{\pi}{2}\right\}$. Multiplication by such a twisting factor is equivalent to swapping the imaginary and real parts of the complex modulated symbol. A further increase in the number of phase quantization levels does not lead to a significant effect. Therefore, it is recommended to use a one-bit control command.
Fig. 5 shows similar dependences of FER on $\mathrm{Eb} / \mathrm{No}$ for MIMO $4 \times 2$ and QO-STBC with a coding rate of 2 ( 4 symbols per 2 clock cycles) with a space-time matrix (13).
For this configuration, it can be seen that even in the case of using a one-bit command, a gain of 2.5 dB is provided.

## VI. Conclusion

In the study, we have shown that for a $4 \times 1$ MIMO configuration, it is possible to completely orthogonalize the quasiorthogonal space-time code by rotating the phases of the two transmitting antennas by the same value depending on the channel factors. In this case, a maximum diversity order of 4 can be obtained with one receive antenna.

The calculation of these phase values is carried out on the receiving side, i.e., at the mobile station, and only one real number is transmitted over the reverse control channel. Simulations performed for this configuration ( $4 \times 1$ ) have shown that just a one-bit phase command is sufficient to obtain a maximum gain of 1.4 dB in comparison to a similar openloop system. For a $4 \times 2$ MIMO and QO-STBC configuration with a coding rate of $2(4 \times 2)$, the gain using a 1 -bit control command is 2.5 dB .

The proposed space-time diversity method on the transmitting side is a combination of quasi-orthogonal space-time coding and feedback. It is important that the structure of the
receiver and transmitter does not change, i.e., the proposed method is universal. In addition, the control of the phases of the transmitting antennas used in this method does not change the power of the emitted signal, which does not lead to a deterioration in the interference environment.

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