Analysis of Capacity of Picocell with Dominating Video Streaming Traffic

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Saint-Petersburg, 2014

A common problem for a network developer

- Centralized wireless networks may contain a sufficient number of users.
- If resources are not enough for requiring to quality of service (QoS) for each user, then network is congested.
- The possible causes of users video playback degradation:
  - **Rebuffering** – state of streaming invoked when the playback buffer is emptied.
  - **Jitter** – variation of playback speed.
  - **Playback smoothness** – frequency of video bit rate switching.
- **A common problem for a network developer is estimation of number of users, who can simultaneously watch video content without playback degradation.**

General description of system model

- Number of active users is constant N.
- All the users request videos in the same bit rate:

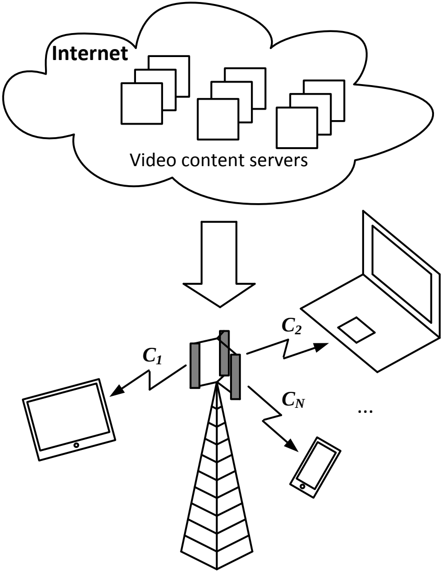
$$R_i = R$$

- Maximum channel throughput is calculated by the Shannon formula:

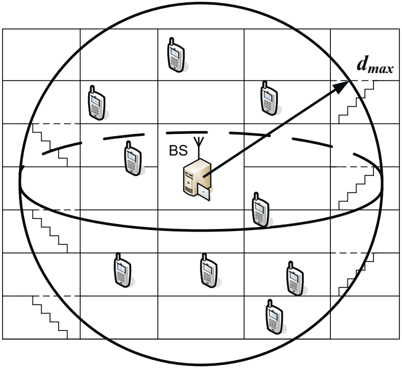
$$C_i = \Delta F \log_2(1 + q_i)$$

- User SNR depends on a distance between BS and UE:

$$q_i = \frac{a}{d_i^{\gamma}}$$



- User disposition can be modeled with an uniform distribution in sphere, which radius corresponds to a maximum distance d_{max}.



CDF:

$$F_d(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{x^3}{d_{max}^3}, & \text{if } 0 \le x \le d_{max} \\ 1, & \text{if } x > d_{max} \end{cases}$$

PDF:

$$f_d(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{3x^2}{d_{max}^3}, & \text{if } 0 \le x \le d_{max} \\ 0, & \text{if } x > d_{max} \end{cases}$$

Congestion and network capacity

Definition

Congestion is an event, when total amount of required resources is greater than one.

$$Pr\{\text{Congestion}\} = Pr\left\{\sum_{i=1}^{N} \frac{R_i}{C_i} \geq 1\right\} = Pr\left\{\sum_{i=1}^{N} \frac{1}{\log_2(1+q_i)} \geq \frac{\Delta F}{R}\right\}$$

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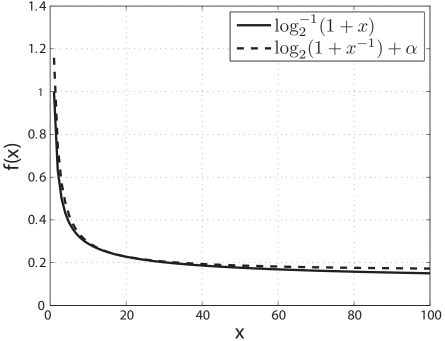
Definition

Network capacity N_c is a maximum number of users, for which the probability of congestion is less than given level p_c.

$$N_c = \arg\max_{N}\left\{Pr\left\{\sum_{i=1}^{N}\frac{R_i}{C_i} \geq 1\right\} \leq p_c\right\}$$

Auxiliary Inequality

$$\frac{1}{\log_2(1+x)} \leq \log_2\left(1+\frac{1}{x}\right)+\alpha$$



$$Pr\{\text{Congestion}\} \leq Pr\left\{\sum_{i=1}^{N}\left(\log_2\left(1+\frac{1}{q_i}\right)+\alpha\right) \geq \frac{\Delta F}{R}\right\}$$

Approximate Calculation of Congestion Probability

1. Denote: $\log_2\left(1+\frac{1}{q_i}\right)+\alpha$ as X_i and $\sum\limits_{i=1}^{N} X_i$ as S_N.
2. According to Central Limit Theorem (CLT), distribution of S_N is close to the normal distribution with mean $E\left[S_N\right]$ and variance $Var\left[S_N\right]$.

$$Pr\left\{\text{Congestion}\right\} \leq Pr\left\{S_N \geq \frac{\Delta F}{R}\right\} \approx Q\left(\frac{\Delta F - E\left[S_N\right]R}{R\sqrt{Var\left[S_N\right]}}\right)$$

3. Here: $E\left[S_N\right] = N \cdot E\left[X_i\right]$, $Var\left[S_N\right] = N \cdot Var\left[X_i\right]$, since X_i are independent random variables.

Upper Bound of Congestion Probability

1. As was mentioned above:

$$Pr\{\text{Congestion}\} \leq Pr\left\{S_N \geq \frac{\Delta F}{R}\right\}$$

2. For finding upper bound for $Pr\left\{S_N \geq \frac{\Delta F}{R}\right\}$ Hoeffding inequality can be used. According to it:

$$Pr\left\{S_N - E[S_N] \geq t\right\} \leq \begin{cases} e^{-\frac{2t^2}{N(x_{max}-x_{min})^2}}, & t > 0 \\ 1, & t \leq 0 \end{cases},$$

where $X_i \in [x_{min}, x_{max}]$, $x_{min} = \alpha$, $x_{max} = \log_2\left(1 + \frac{d_{max}^3}{a}\right) + \alpha$.

3. Thus:

$$Pr\{\text{Congestion}\} \leq \begin{cases} e^{-\frac{2}{N}\left(\frac{\frac{\Delta F}{R} - N \cdot E[X_i]}{\log_2\left(1+\frac{d_{max}^3}{a}\right)}\right)^2}, & N < \frac{\Delta F}{R \cdot E[X_i]} \\ 1, & \text{otherwise} \end{cases}$$

Network Capacity Estimation

- **Approximate value** of network capacity, **based on CLT**:

$$N_c \approx \left(\frac{-g_2 + \sqrt{g_2^2 - 4 g_1 g_3}}{2 g_1} \right)^2$$

- **Lower bound** for network capacity, **based on Hoeffding inequality**:

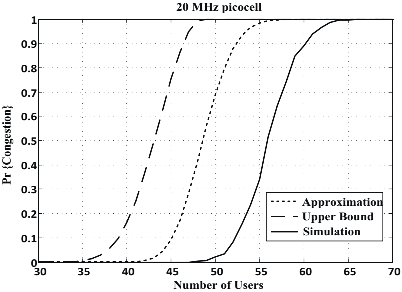
$$N_c \geq \left(\frac{-g_4 + \sqrt{g_4^2 - 4 g_1 g_3}}{2 g_1} \right)^2$$

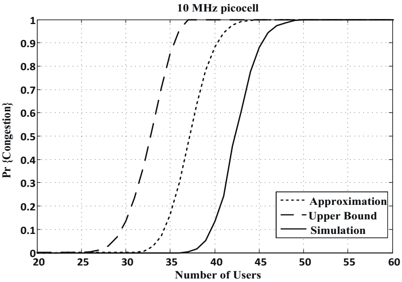
Where:

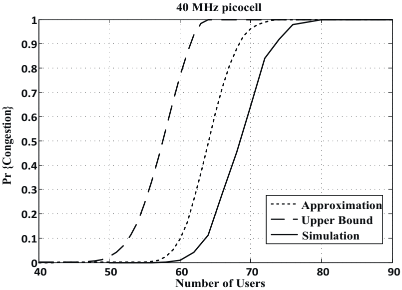
$$\begin{cases} g_1 = E[X_i] \\ g_2 = \sqrt{Var[X_i]} Q^{-1}(p_c) \\ g_3 = -\Delta F R^{-1} \\ g_4 = \log_2 \left(1 + \frac{d_{max}^3}{a}\right) \sqrt{-\frac{1}{2} \ln p_c} \end{cases}$$

Numerical example

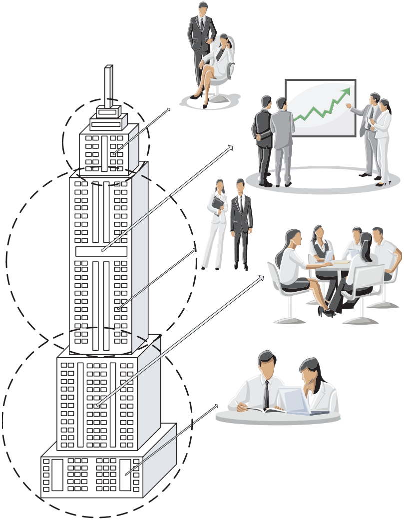
- Bandwidth $\Delta F \in \{10MHz, 20MHz, 40MHz\}$
- Carrier frequency $2GHz$
- UE noise figure $N_f = 10$
- Video bit rate $R = 500kbps$
- Maximum distance $d_{max} = 60m$



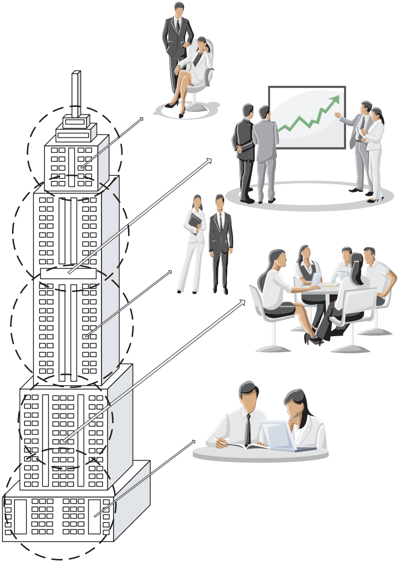




- Three power picocells



- Five weak picocells



Conclusion

Results:

- Congestion probability for wireless video streaming picocell network was investigated.
- Convenient majorant of function $\frac{1}{\log_2(1+x)}$ was proposed.
- Proposed expressions allows simple estimating of network capacity. However the results are applicable only for environments, where path loss factor is close to «3».

Further research:

- Generalization of obtained results for wider conditions may be a direction of further research.

Auxiliary calculations

1. Calculation of X_i mean:

$$E\left[X_i\right] = \int\limits_0^{d_{max}} \left[\log_2\left(1+\frac{x^3}{a}\right)+\alpha\right] f_d(x)dx$$

$$E\left[X_i\right] = \frac{1}{d_{max}^3}\left[\int\limits_0^{d_{max}^3} \log_2\left(1+\frac{t}{a}\right)dt + \int\limits_0^{d_{max}^3} \alpha dt\right] = \frac{k\ln m - 1}{\ln 2} + \alpha,$$

where $t = x^3$, $m = 1 + \frac{d_{max}^3}{a}$ and $k = 1 + \frac{a}{d_{max}^3}$.

2. Calculation of X_i variance:

$$Var\left[X_i\right] = E[X_i^2] - E\left[X_i\right]^2$$

$$E\left[X_i^2\right] = \frac{k(\ln m - 1)^2 - k}{(\ln 2)^2} + 2\alpha E\left[X_i\right] + \alpha^2$$