Detection of Primary User in Cognitive Radio Network

Lu Wei and Olav Tirkkonen
Helsinki University of Technology (TKK)

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Outline

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- It is essential that the secondary user will make a quick and reliable decision based on spectrum sensing.
- Recently emerged eigenvalue-based detection is promising method to solve this problem.

Signal Model

- Assume we have K sensors and N samples, the received $K \times N$ data matrix $\mathbf{Y}$ is

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$$\mathbf{H_1} \quad : \quad y_k^{(n)} = h_k^{(n)} s_k^{(n)} + n_k^{(n)}, \tag{3}$$

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Threshold Optimization

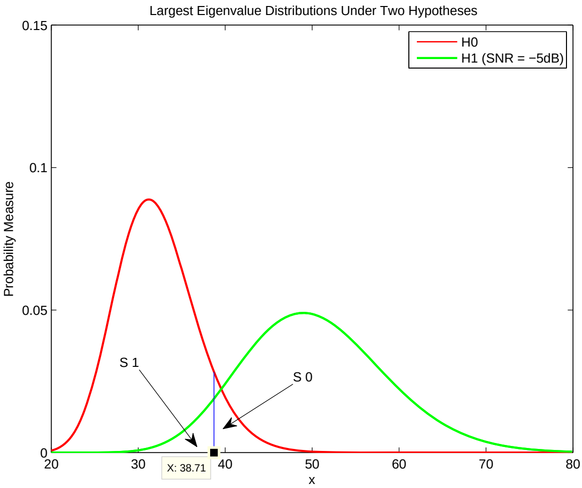

Figure: $K = 4,\ N = 20,\ C_0 = 1,\ C_1 = 1.5.$

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Fixed SNR

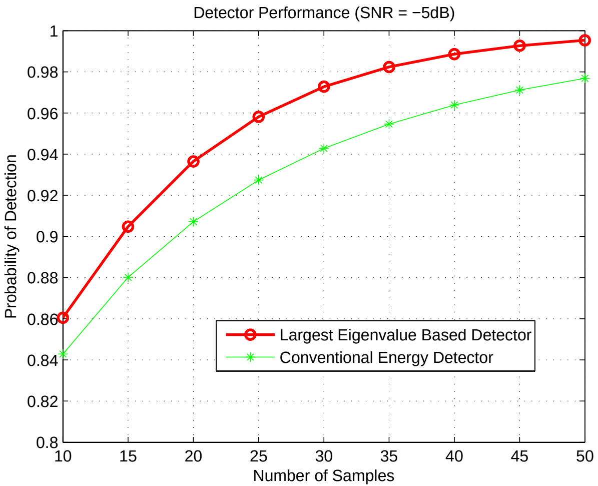

Figure: Performance Comparison

Fixed Sample Size

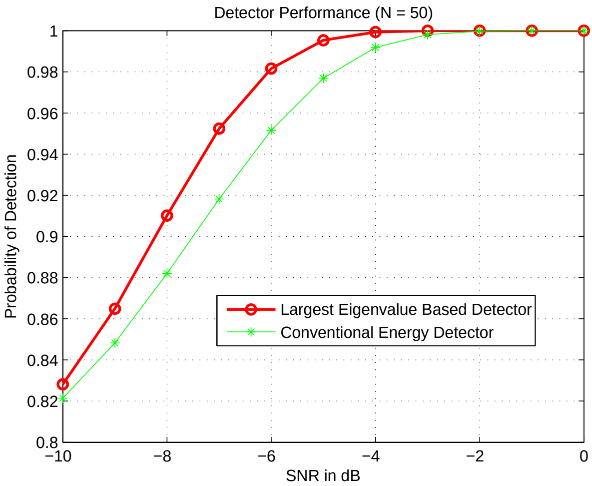

Figure: Performance Comparison

Thank you!