HOLOGRAPHIC IMPLEMENTATION OF A LINEAR PREDICTOR OF RANDOM PROCESSES: INFLUENCE OF HIGH-PASS AND LOW-PASS FILTERING ON THE PROCESS CHARACTERISTICS

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Application of prediction

Task of prediction is actual for telecommunication networks. It allows the usage of the telecommunication nodes devices to be optimized.

Traffic in telecommunication networks is described by the model of random process with stationary increments, particularly, Fractal Brownian Motion*, so the model of linear predictor can be applied for its stationary increments.

<u>Advantages of optic realization:</u>

- 2D data can be processed in parallel
- The computations are carried out by light speed
- learning of the predictor is implemented in real time

W.E.Leland, M.S.Taqqu, W.Willinger, and D.V.Wilson. On the self-similar nature of Ethernet traffic (extended version).IEEE/ACM Transactions of Networking, 2(1):1-15,1994,

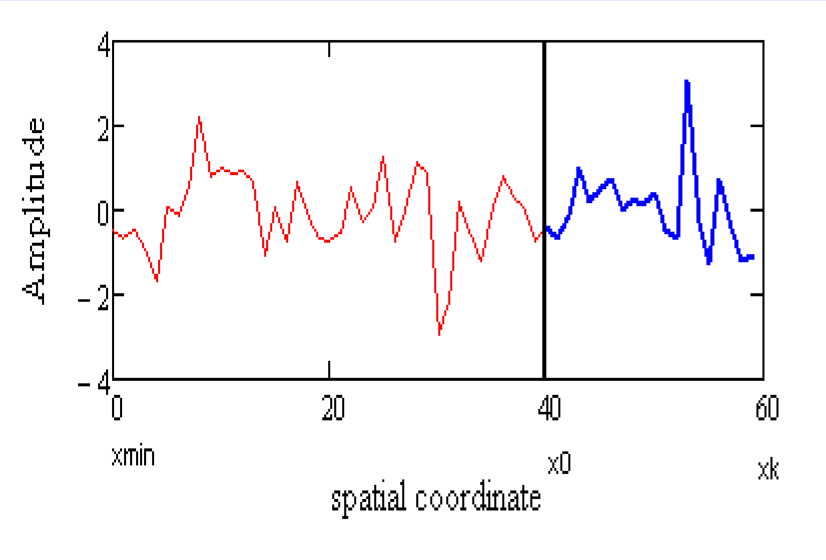
Real-time estimation of the parameters of long-range dependence, IEEE/ACM Transactions on Networking (TON) , Volume 8, Issue 4, 2000

Model of linear predictor of random processes

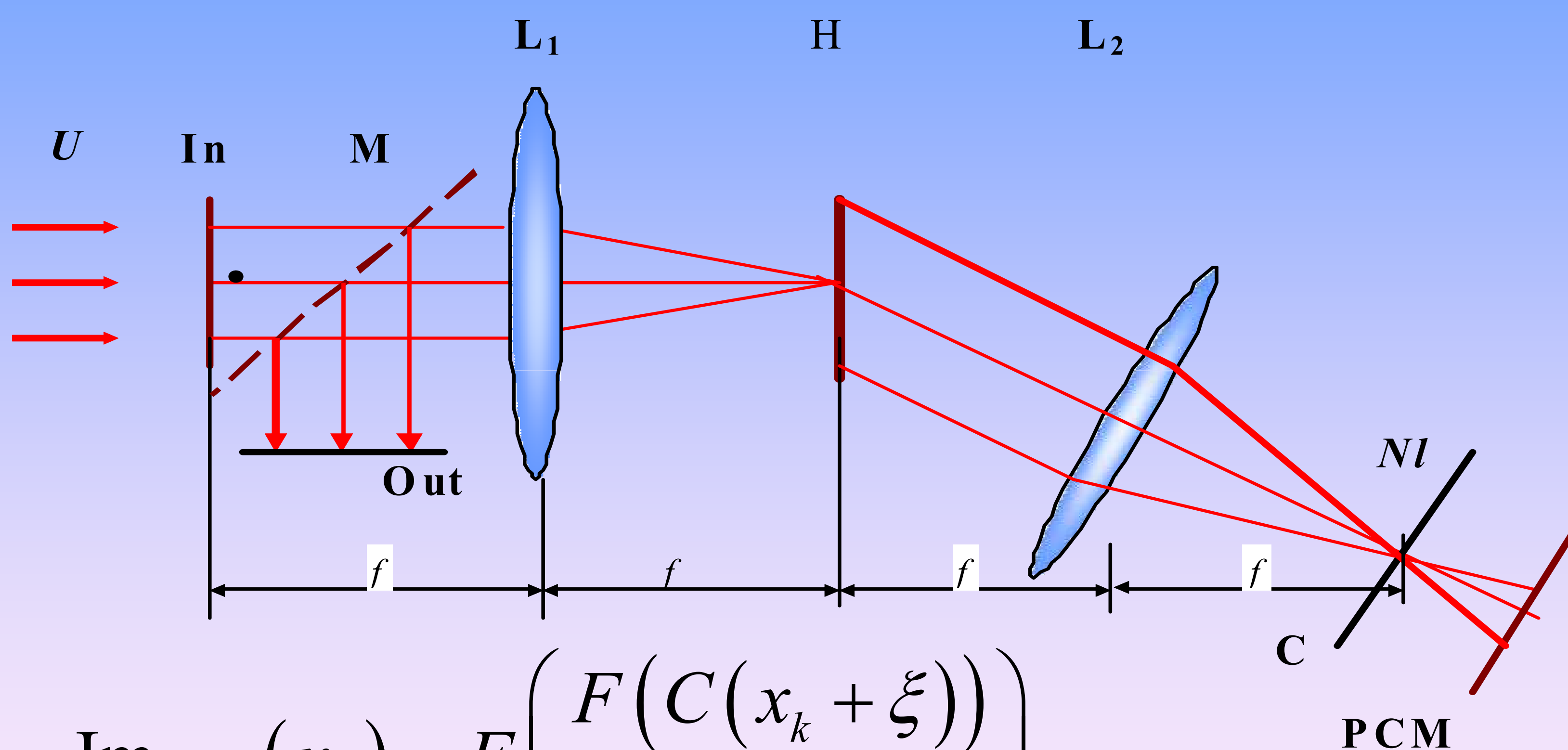
- Task of prediction is particular case of best estimate task
- Model of the predictor is based on the linear regression
- Evaluates the estimation of the mean of process in unobserved point

$$Im_{Bpred}(x_k) = \int_{x_{Min}}^{x_0} Im_B(x_0 - x)a(x)dx$$

$$\int_{x_{Min}}^{x_0} a(x)C(|x-\xi|)dx = C(x_k + \xi)$$



Scheme of Fourier – holography with wave front reversal



$$\mathrm{Im}_{pred}\left(x_k\right) = F\left(\frac{F\left(C\left(x_k+\xi\right)\right)}{F^*\left(\mathrm{Im}\left(x\right)\right)}\right)$$

Conditions of the correct prediction

- Zero mean
- Stationarity of the process

<u>The problem is non-stationarity of majority of processes</u>

The approach to the task solving is based on the limitedness of dynamic range of the recording medium, due to this frequency filtering always appears.

Conditions of the correct prediction

Zero mean

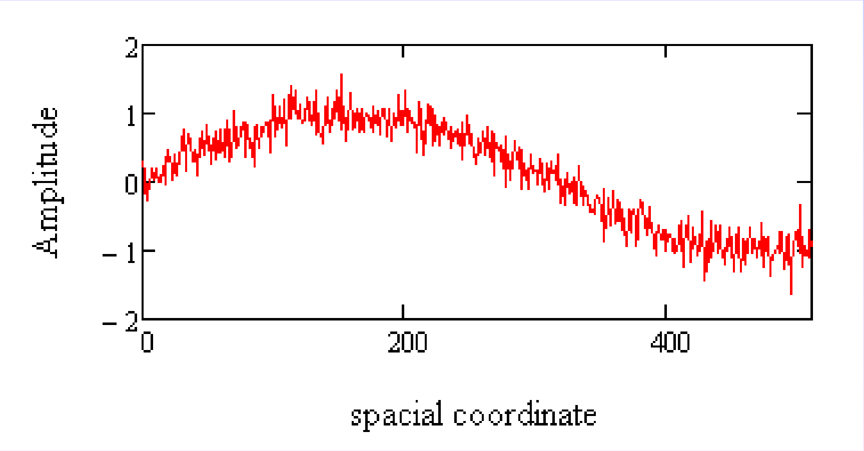
- Frequency filtering can be applied (rejection of zero frequency)

Stationarity of process

- Non-stationarity of dispersion may be caused by high-frequency (“white”) noise →low-pass filtering can be applied to improve stationarity
- Non-stationarity of mean is caused by low frequencies (process mean is not constant) →dividing of process on the stationary intervals

Improvement of stationarity

- L/Tmin - integer, L – length of process, Tmin – period of lowest frequency of spectrum →process is stationary
- L/Tmin is not integer → process is not stationary
- To reject this frequency high-pass filtering can be applied to improve stationarity

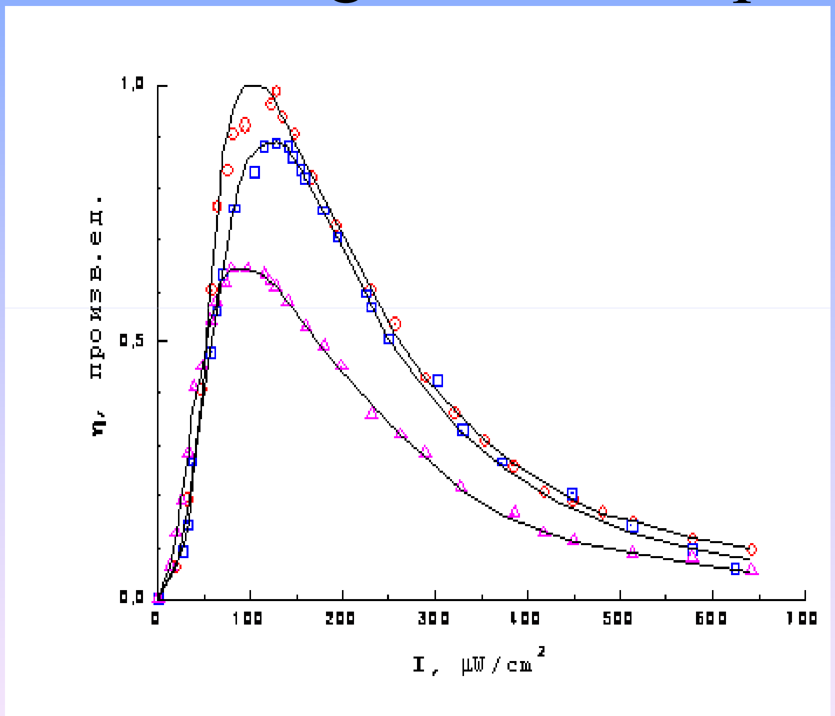


Non-stationarity of process mean: L/Tmin is not integer

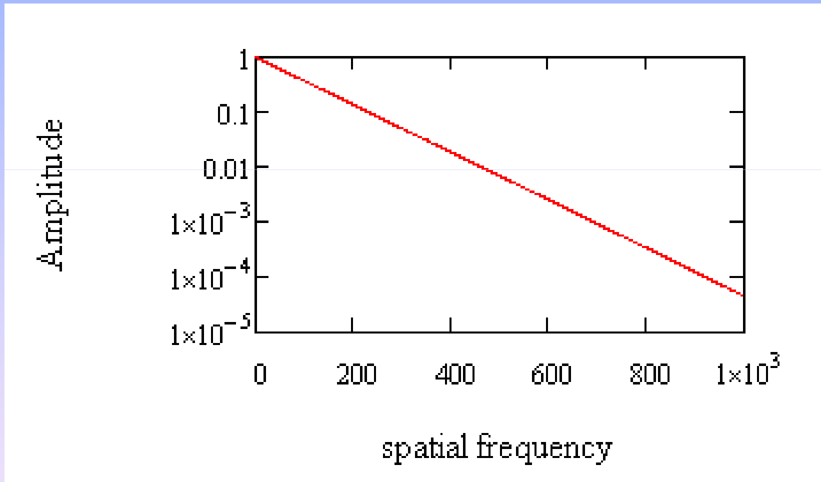
Limitedness of dynamical range

Limitedness of dynamical range

→filtering in Fourier-space



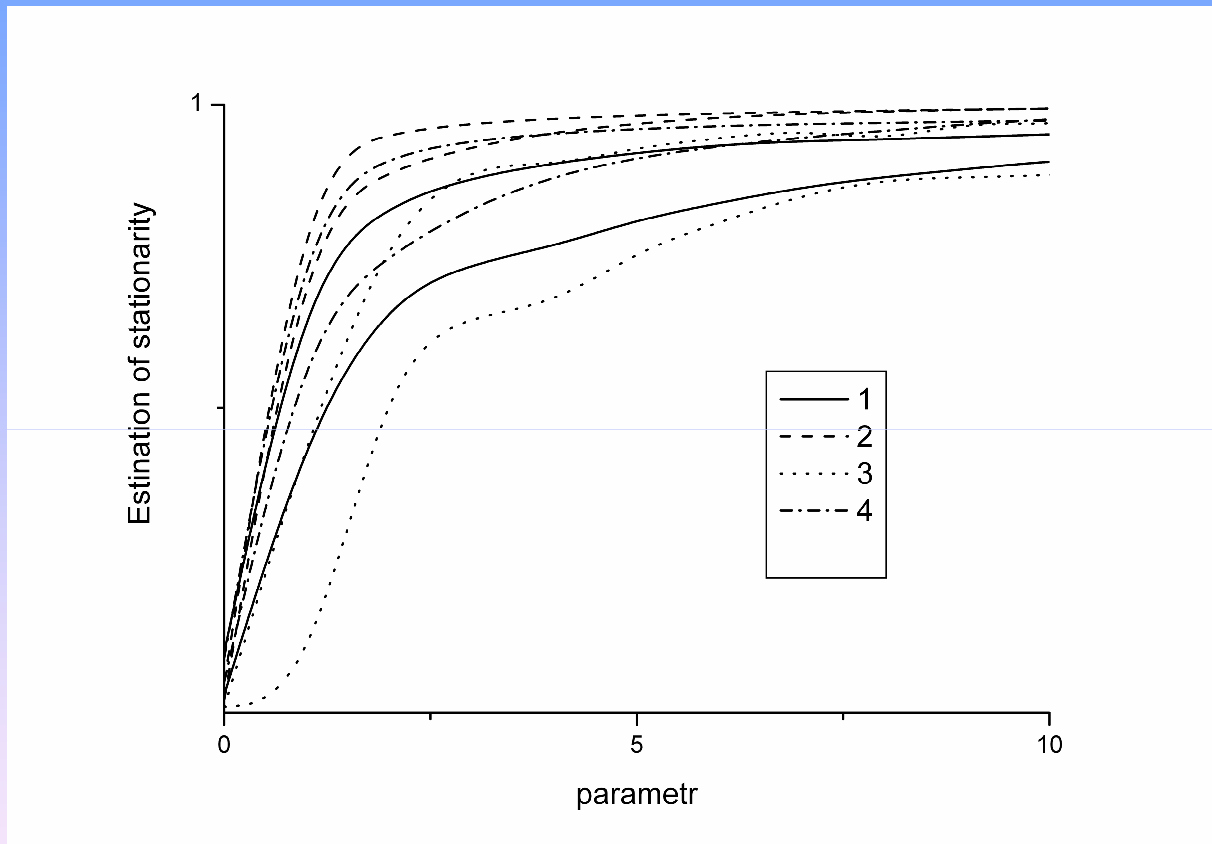
Dependence of local diffraction efficiency from exposure for the great number of recording media



Fourier-spectrum

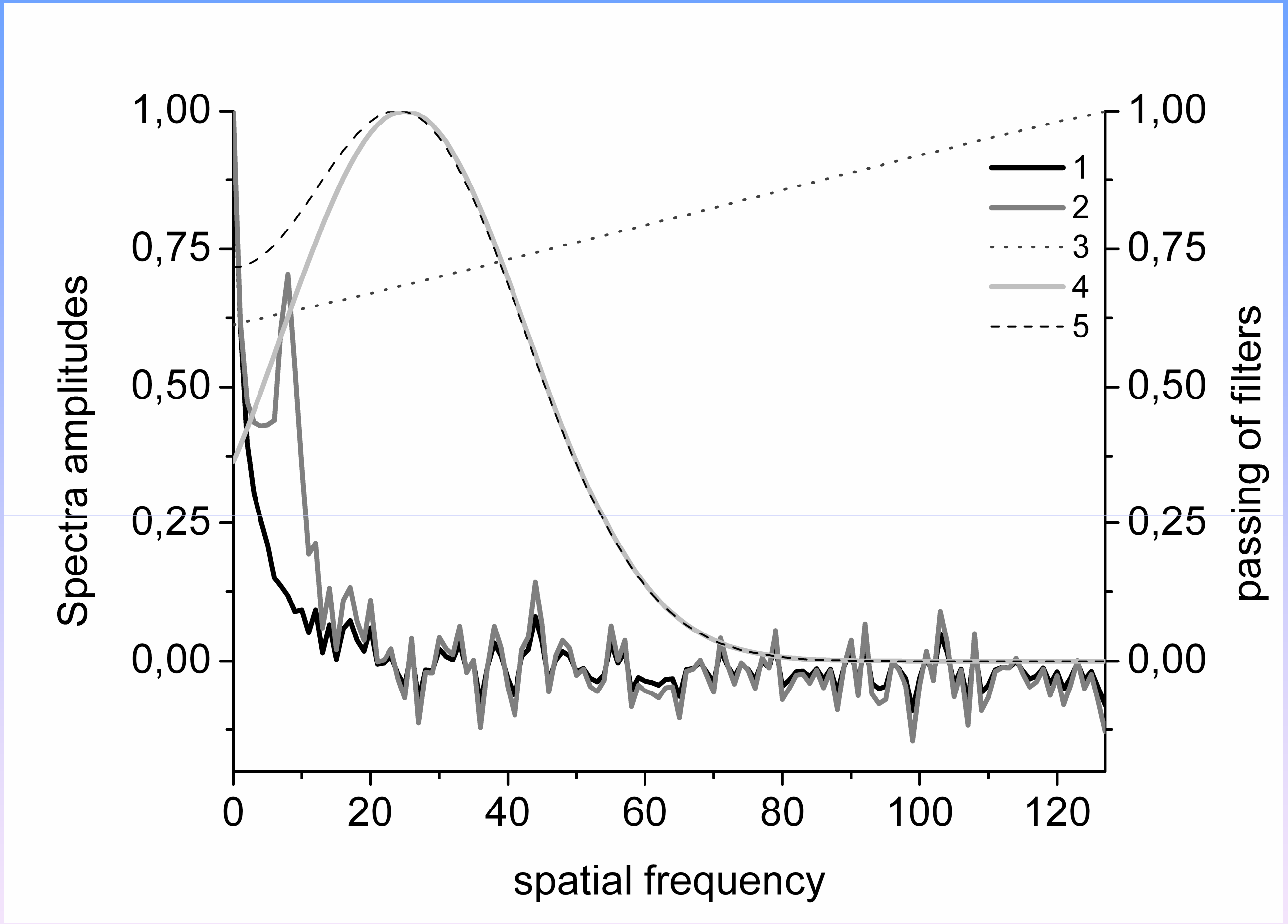
$$F(\nu) = \exp(-\tfrac{\nu}{100})$$

Improvement of stationarity after low-pass filtering



Estimations of stationarity of process (which has non-stationarity of dispersion) after filtering: 1 – moving average filter, 2 – rectangle, 3 – exponential, 4- Gaussian

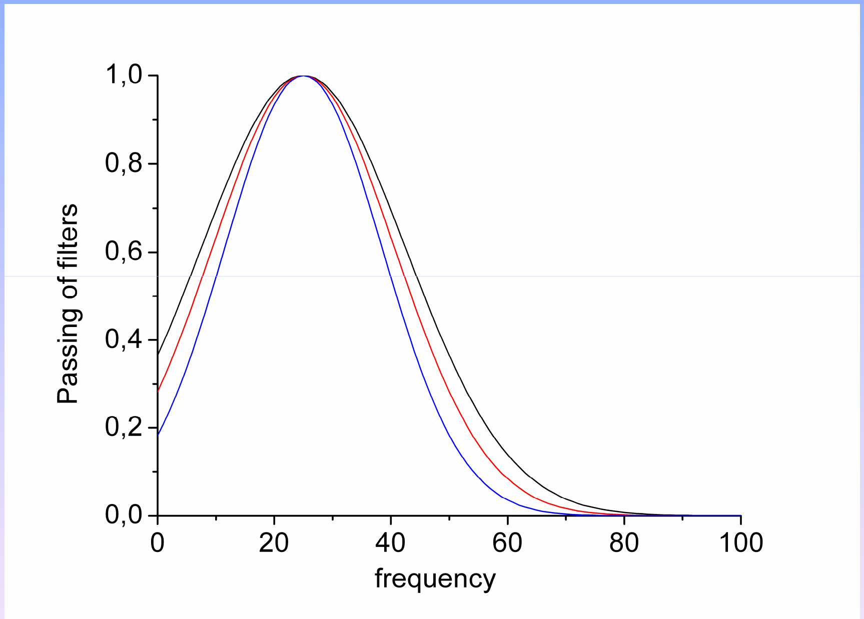
Spectra and filters

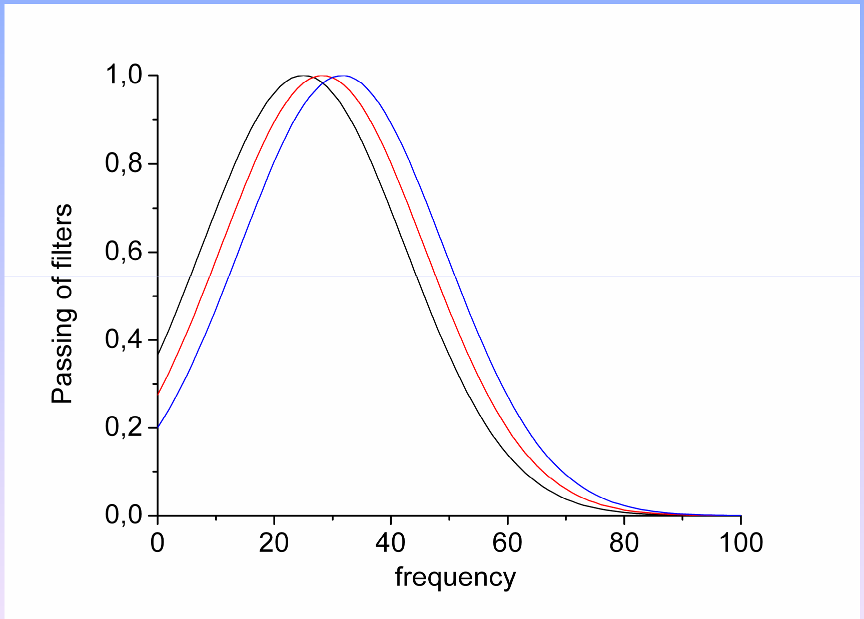


Spectra of processes (1- exponential, 2 – with local maximum);

filters (3 – sigmoid, 4 – gaussian, 5 – sum of two gaussians)

High-pass filtering

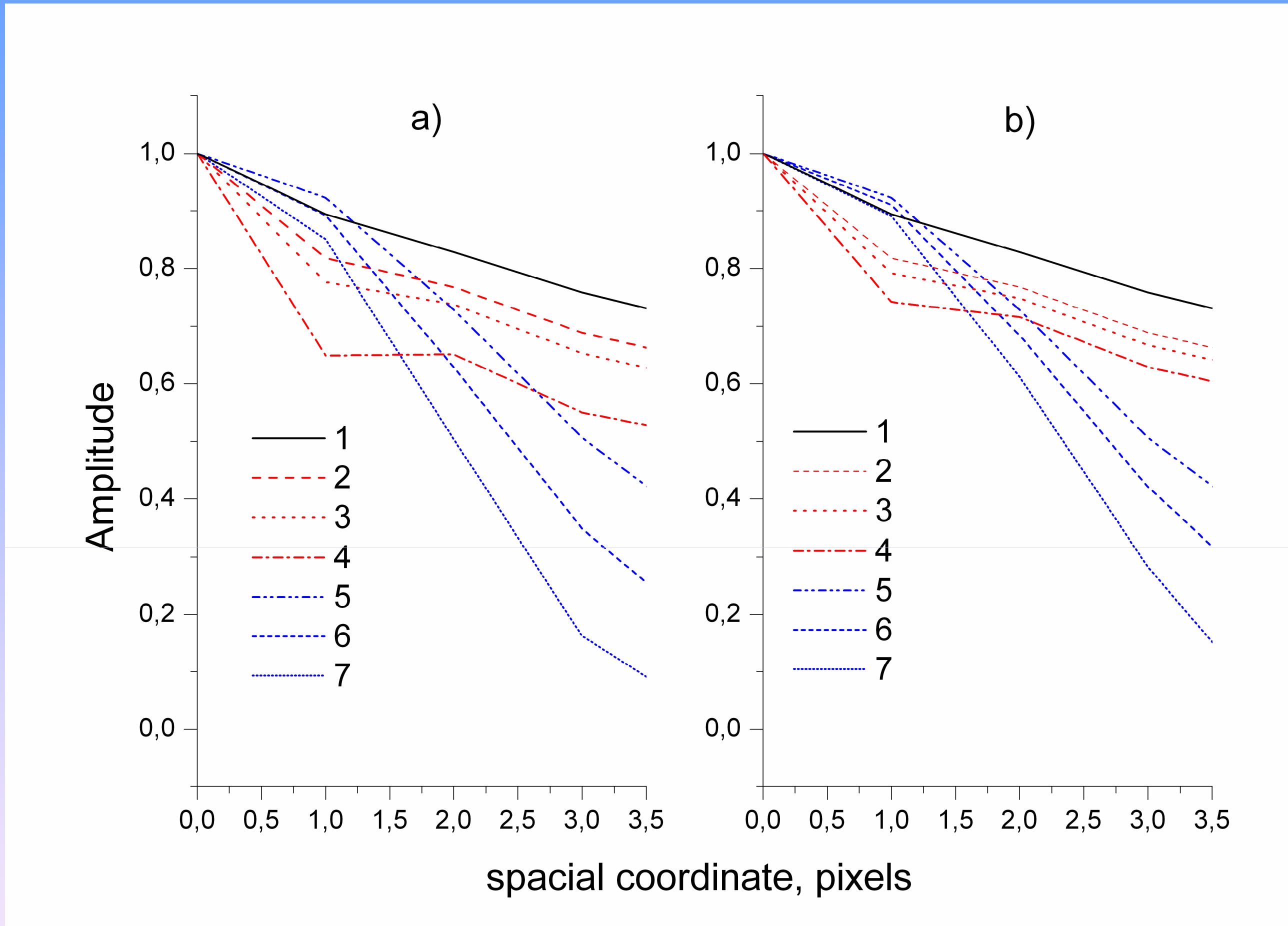




Narrowing of filter

Shift of filter

Section of correlation functions



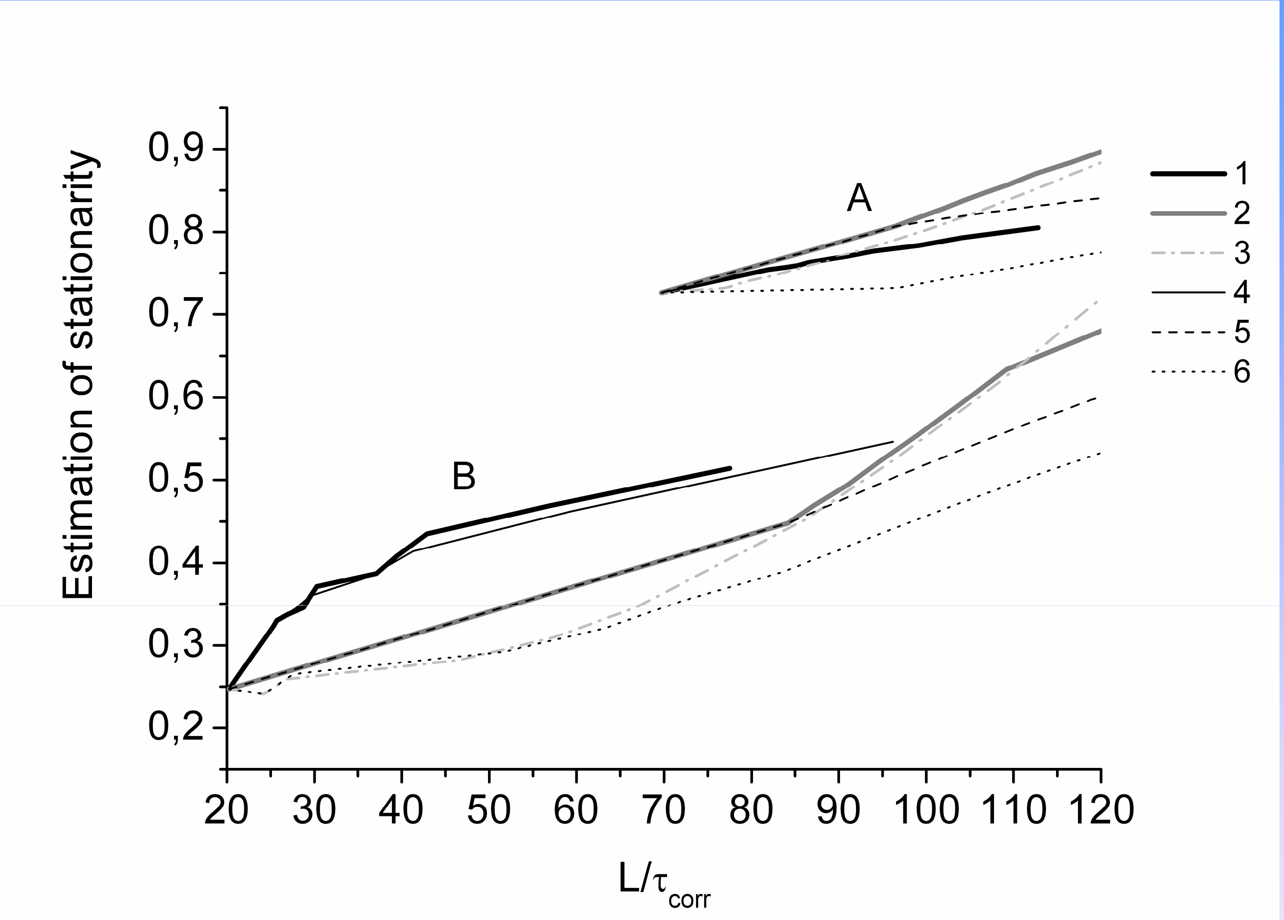
a) shift of filter maximum to the area of high frequencies; b) shift of filter; 1 –non-filtered, 2, 3, 4 –sigmoid filter; 5,6,7 –gaussian filter

Formula for stationarity estimation

$$\mathrm{Est}\left(\frac{L}{\tau_{corr}}\right) = 1 - \left(\frac{(mean(left) - mean(right))^2}{var(real)}\right),$$

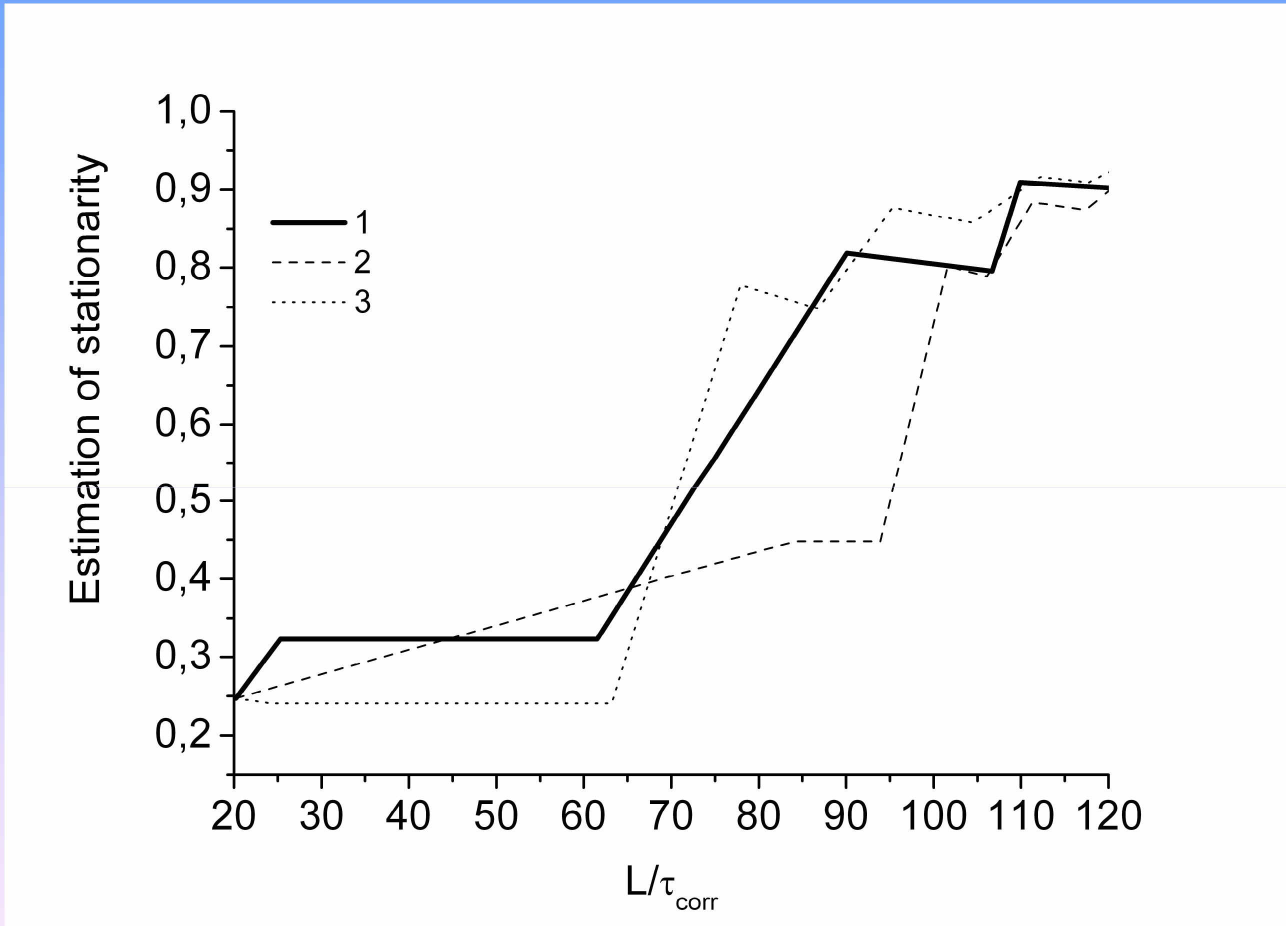
L – length of process, $\boldsymbol{\tau_{corr}}$ – correlation radius, ***mean*** - process mean, ***left*** and ***right*** – parts of process of equal length, **var –** dispersion of process, ***real*** – process

Estimation of stationarity



Estimations of stationarity after filtering by the narrowing of functions (group A – spectrum with local maximum; group B– exponential spectrum): 1 – sigmoid filter, 2 – gaussian filter, 3 – sum of two gaussians; filtering by shift of functions: 4 – sigmoid, 5 – gaussian, 6 – sum of two gaussians

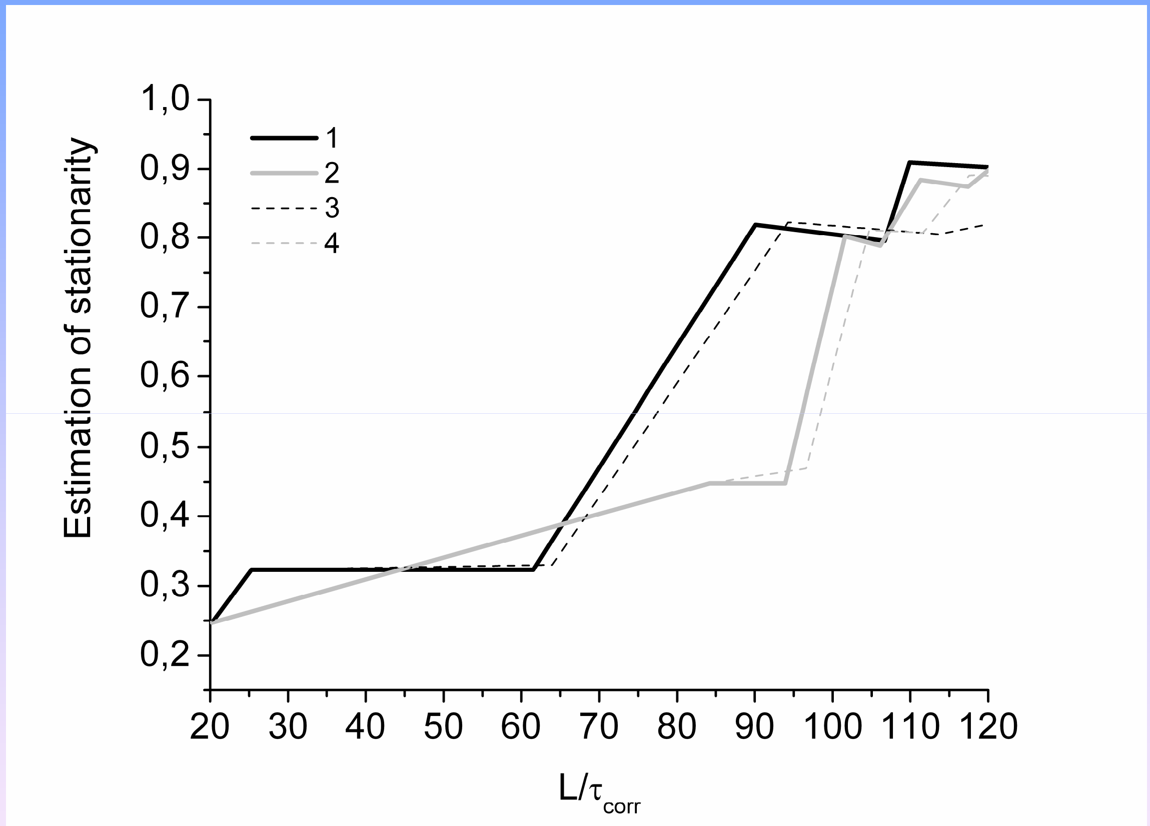
High-pass filtering



Estimations of stationarity of exponential spectrum after rejection: 1 – sigmoid filter, 2 – gaussian filter, 3 – sum of two gaussians

High-pass filtering

Combination of rejection and narrowing of filters



1, 3 –rejection of sigmoid and gaussian filters accordingly;

2, 4 – rejection and narrowing of sigmoid and gaussian filters

Conclusion

Thus, limitedness of the dynamical range of the holographic recording media for the improvement of stationarity of processes by the method of high-pass filtering can be applied if non-stationary is caused by the low frequencies. It is shown that low frequencies had a prevailing influence on the estimation of stationarity of process used in the model of predictor.

Thank you for your attention!